Quasiparticles in EFT

(with: H.-W. Hammer, Ulf-G. Meißner)

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Introduction

Why dilute Fermi systems?

- traditional benchmark for new approaches to many-body physics
- first step towards EFT description of nuclear matter
- cold Fermi gases are current field of research:
- controlled expansion in k_F/Λ

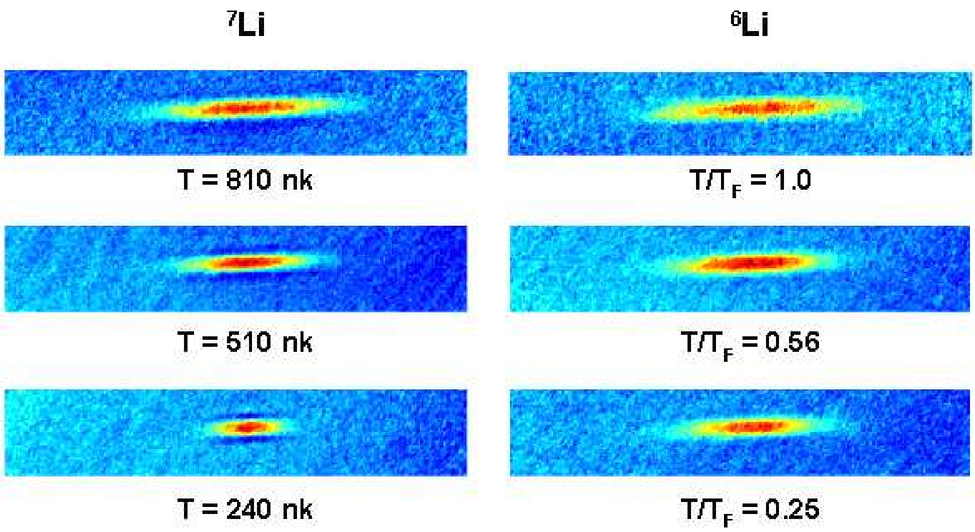


Figure 1: Hulet et al.

Two-body EFT in the vacuum

(following: Hammer, Furnstahl, Nucl. Phys. A 678 (2000) 277)

- Two-particle scattering Amplitude:

$$T(k, \cos\theta) = \frac{4\pi}{M} \sum_l \frac{2l+1}{k \cot \delta_l - ik} P_l(\cos\theta)$$

- Effective range expansion for the scattering amplitude with $a_s \sim a_p \sim r_s \sim 1/\Lambda$ and $k \ll \Lambda$

$$\begin{aligned} T(k, \cos\theta) &= -\frac{4\pi a_s}{M}\left[1 - i a_s k + (a_s r_s/2 - a_s^2) k^2\right] \\ &\quad -\frac{4\pi a_p^3}{M} k^2 \cos\theta + \mathcal{O}(k^3/\Lambda^3)\ . \end{aligned}$$

→ Goal: reproduce this with an appropriate EFT

⋆ $k \ll \Lambda \Rightarrow$ all interactions appear pointlike.

⋆ we can use contact interactions in our EFT

- Construct the most general Lagrangian:

$$\mathcal{L} = \psi^\dagger \left[i\partial_t + \frac{\overrightarrow{\nabla}^2}{2M} \right] \psi - \frac{C_0}{2} (\psi^\dagger \psi)^2 + \frac{C_2}{16} \left[(\psi\psi)^\dagger (\psi \overleftrightarrow{\nabla}^2 \psi) + \text{H.c.} \right] + \ldots .$$

⋆ relativistic corrections can be added systematically

→ Now: determine the C_{2i} (matching)

What about Divergences?

- Loop integrals that appear for two body-scattering within this EFT:

$$L_n = \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{q^{2n}}{k^2 - q^2 + i\epsilon}$$

- Apply cutoff Λ_C

$$L_0 \to -\frac{1}{2\pi^2}\Lambda_c - \frac{i}{4\pi}k + \mathcal{O}(k^2/\Lambda_c) \ .$$

$\longrightarrow$ power divergences !

- Dimensional regularization with minimal subtraction removes these divergences!

$\longrightarrow$ power divergences are subtracted

$\longrightarrow$ no other divergences appear to the order we do calculations

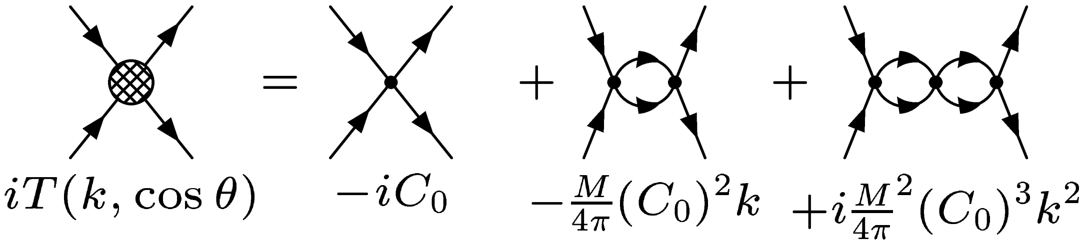
$\longrightarrow$ loop integrals only pick up the scale of the external momentum

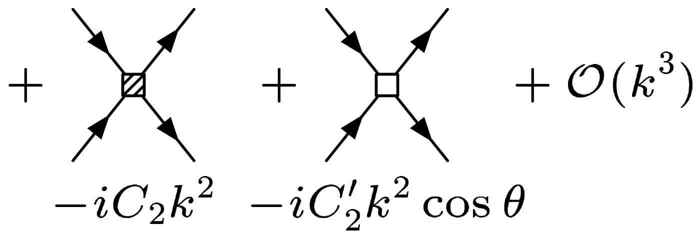
$$\text{e.g.} \qquad L_0 \longrightarrow -\frac{i}{4\pi}k$$

⇒ We now have a power counting:

- for every propagator a factor of M/k^2
- for every loop integration a factor k^5/M
- for every n-body vertex with $2i$ derivatives a factor $k^{2i}/(M\Lambda^{2i+3n-5})$

Matching:





- Compare with the effective range expansion for the scattering and write the C_{2i} in terms of the effective range parameters:

$$C_0 = \frac{4\pi a_s}{M}$$

$$C_2 = C_0 \frac{a_s r_s}{2}$$

$$C_2' = \frac{4\pi a_p^3}{M}$$

The Dilute Fermi System

Consider:

- Ground state of a gas of fermions at $T = 0$
- Interaction between particles is repulsive
  $\Rightarrow$ No Cooper-pairing $\rightarrow$ no BCS instability !
- $k_F a_s \ll 1 \Rightarrow$ low density
  $\Rightarrow$ will be used as expansion parameter

The Proper Self-Energy

- Dysons's Equation:

$$G(\tilde{p})_{\alpha\beta} = G^0_{\alpha\beta}(\tilde{p}) + G^0_{\alpha\lambda}(\tilde{p})\Sigma^*_{\lambda\mu}(\tilde{p})G_{\mu\beta}(\tilde{p})$$

$\rightarrow$ for spin-independent interactions:

$$G_{\alpha\beta} = \frac{1}{p_0 - p^2/(2M) - \Sigma^*(\tilde{p})}\delta_{\alpha\beta}$$

$\Rightarrow$ The pole of the full Green's function is determined by:

$$p_0 - \frac{p^2}{2M} - \Sigma^*(\tilde{p}) = 0 \qquad (\star)$$

with $\quad p_0 = \epsilon_p + i\gamma_p$

ϵ_p: quasiparticle excitation energy
γ_p: quasiparticle width

- Compute Σ^* perturbatively to solve for $(\star)$

Transition to the medium:

- apply adapted power counting from the vacuum theory
  - ⋆ internal momenta are of order k_F due to the regularisation scheme used
  - ⋆ the external momentum will be kept at the order of k_F (quasiparticles are only well-defined quantities near the Fermi surface)

  $\Rightarrow$ thus, use the following power counting:
  - · for every propagator a factor of M/k_F^2
  - · for every loop integration a factor k_F^5/M
  - · for every n-body vertex with $2i$ derivatives a factor $k_F^{2i}/(M\Lambda^{2i+3n-5})$

  ∗ Note again: $k_F a_s \ll 1$

- modify the Green's function for the medium:

$$iG^0_{\alpha\beta}(\tilde{k}) = i\delta_{\alpha\beta}\left(\frac{\Theta(k-k_F)}{k_0-\omega_k+i\epsilon}+\frac{\Theta(k_F-k)}{k_0-\omega_k-i\epsilon}\right)$$

Other Observables

(following Fetter & Walecka)

- Chemical Potential μ
  - minimal energy to add a particle $\rightarrow$ excitation energy at the Fermi surface
- Effective Mass m^*
  - defined via the group velocity of a quasiparticle state at the Fermi surface; that is the slope of the excitation energy at $p = k_F$:

$$m^* = k_F \left(\left. \frac{\partial \epsilon_p}{\partial p} \right|_{k_F} \right)^{-1}$$

- Energy Density E/ρ
  - use relation between the chemical potential and energy density:

$$\mu = \left(\frac{\partial E}{\partial N} \right)_V \quad \text{at} \quad S = 0$$

  - Integrate

$$\Rightarrow E = \int_0^N \mathrm{d}N' \mu(S = 0, V, N')$$

$$\rightarrow \int_0^N \mathrm{d}N' [k_F(N')]^\lambda = \frac{3}{3+\lambda} k_F^\lambda N$$

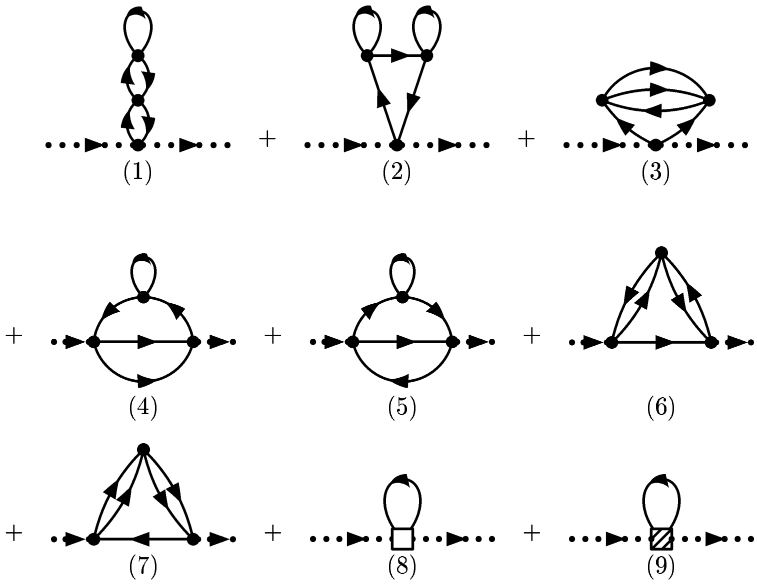
Computation of $\Sigma^*(p)$ at $\mathcal{O}((k_F a_s)^3)$

- Expand proper self-energy in orders of $k_F a_s$

$$\epsilon_k + i\gamma_k = \frac{k^2}{2M} + (k_F a_s)\tilde{\Sigma}^*_{(1)}(k) + (k_F a_s)^2\tilde{\Sigma}^*_{(2)}(p)$$
$$+(k_F a_s)^3\left[\Sigma^*_{(3)}(k) + \Sigma^*_{(1)}\frac{d\Sigma^*_{(2)}(\tilde{k})}{dk_0}\bigg|_{k_0=\frac{k^2}{2M}}\right] + \mathcal{O}((k_F a_s)^4)$$

- third order contribution to $\epsilon_p + i\gamma_p$

$$\epsilon_3 + i\gamma_3 = \Sigma^*_{(3)}(k) + \Sigma^*_{(1)}\frac{d\Sigma^*_{(2)}(\tilde{k})}{dk_0}\bigg|_{k_0=\frac{k^2}{2M}}$$

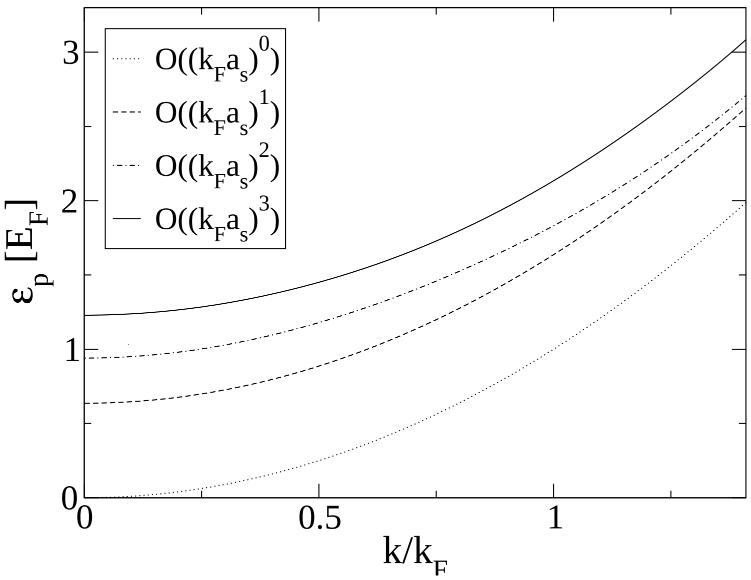


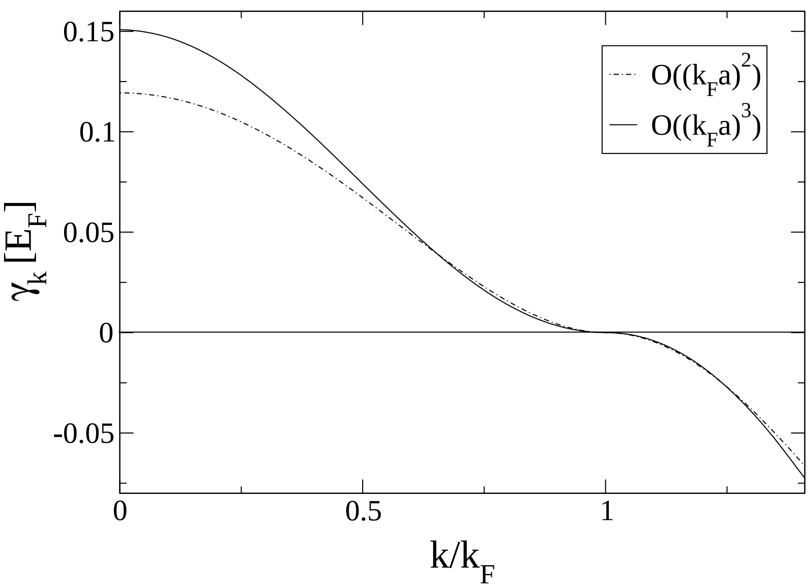
Diagrams contributing to Σ^* at $\mathcal{O}((k_F a_s)^3)$

Comments:

- diagrams 1 to 3 vanish when evaluated
- diagrams 4 and 5 cancel the derivative term
  $\rightarrow$ no off-shell dependence !
- diagrams 5 and 6 were simplified analytically and then computed numerically
- diagrams 7 and 8 give purely real results

Results





- exemplified for $k_F a_S = 0.3$ and $g = 4$

Results I:

$$\text{Re}\,\Sigma^*_{3(6)}(\frac{k^2}{2M},k) = \frac{k_F^2}{2M}(g-1)(g-3)\frac{2^3}{\pi^3}(k_F a_s)^3\left\{0.5908-0.8829(1-v)\right.$$
$$+0.2140(1-v)^2+3.7284(1-v)^3-2.7392(1-v)^4$$
$$\left.-4.7509(1-v)^5+7.0977(1-v)^6-2.6709(1-v)^7\right\},$$

$$\text{Re}\,\Sigma^*_{3(7)}(\frac{k^2}{2M},k) = \frac{k_F^2}{2M}(g-1)\frac{2^3}{\pi^3}(k_F a_s)^3\left\{0.7811+1.0589(1-v)\right.$$
$$+0.4160(1-v)^2+1.9942(1-v)^3-3.1936(1-v)^4$$
$$\left.-4.8352(1-v)^5+9.6964(1-v)^6-4.1846(1-v)^7\right\},$$

$$\text{Re}\,\Sigma^*_{3(8)}(\frac{k^2}{2M},k) = \frac{k_F^2}{2M}\frac{(g+1)}{\pi}(k_F a_p)^3\left\{\frac{1}{5}+\frac{1}{3}v^2\right\},$$

$$\text{Re}\,\Sigma^*_{3(9)}(\frac{k^2}{2M},k) = \frac{k_F^2}{2M}\frac{(g-1)}{2\pi}\,k_F^3 a_s^2 r_s\left\{\frac{1}{5}+\frac{1}{3}v^2\right\},$$

$$\mathrm{Im}\Sigma^*_{3(6)}(\frac{k^2}{2M},k) = \frac{k_F^2}{2M}\frac{16(g-1)(g-3)}{\pi^2}(k_F a_s)^3\Big[\Big\{-0.0065(1-v)^2 - 1.0778(1-v)^3$$

$$+ 5.2913(1-v)^4 - 13.0528(1-v)^5 + 18.7234(1-v)^6$$

$$- 15.4468(1-v)^7 + 6.7036(1-v)^8 - 1.1653(1-v)^9\Big\}\theta(1-v)$$

$$+ \Big\{-0.0700(1-v)^2 - 5.3866(1-v)^3 - 84.0177(1-v)^4$$

$$- 721.4519(1-v)^5 - 3546.7045(1-v)^6 - 9902.6272(1-v)^7$$

$$- 14617.8867(1-v)^8 - 8855.7811(1-v)^9\Big\}\theta(v-1)\Big]\,,$$

$$\mathrm{Im}\Sigma^*_{3(7)}(\frac{k^2}{2M},k) = \frac{k_F^2}{2M}\frac{8(g-1)}{\pi^2}(k_F a_s)^3\Big[\Big\{-0.3835(1-v)^2 + 3.1281(1-v)^3$$

$$- 7.1868(1-v)^4 + 12.1392(1-v)^5 - 16.3114(1-v)^6$$

$$+ 14.4972(1-v)^7 - 7.2813(1-v)^8 + 1.5639(1-v)^9\Big\}\theta(1-v)$$

$$+ \Big\{0.3715(1-v)^2 + 8.0530(1-v)^3 + 77.9691(1-v)^4$$

$$+ 532.4357(1-v)^5 + 2237.8794(1-v)^6 + 5462.8759(1-v)^7$$

$$+ 7100.2592(1-v)^8 + 3791.8271(1-v)^9\Big\}\theta(v-1)\Big]\,,$$

$$\mathrm{Im}\Sigma^*_{3(8)}(\frac{k^2}{2M},k) = \mathrm{Im}\Sigma^*_{3(9)}(\frac{k^2}{2M},k) = 0\,.$$

Results II

- chemical potential

$$\mu_{(3)} = \frac{k_F^2}{2M}\Big[(g-1)\frac{4}{15\pi}(k_F a_s)^2 k_F r_s + (g+1)\frac{8}{15\pi}(k_F a_p)^3 + (g-1)\{0.20+(g-3)0.15\}(k_F a_s)^3\Big] \quad (11)$$

- effective mass

$$\frac{M^*}{M} = \Big(1+(g-1)\frac{8}{15\pi^2}(k_F a_s)^2(1-7\log 2)+\frac{(g+1)}{3\pi}(k_F a_p)^3 + \frac{(k_F a_s)^2 k_F r_s}{6\pi}(g-1)+0.11(g-1)(g-3)(k_F a_s)^3 - 0.15(g-1)(k_F a_s)^3\Big)^{-1} \quad (12)$$

- third order contribution to the energy per particle

$$\left(\frac{E}{N}\right)_{(3)} = \frac{k_F^2}{2M}\Big[(g-1)\frac{1}{10\pi}(k_F a_s)^2 k_F r_s + (g+1)\frac{1}{5\pi}(k_F a_p)^3 + (g-1)\{0.076+(g-3)0.057\}(k_F a_s)^3\Big] \quad (13)$$

⋆ agrees with previous calculations (Efimov, Amusya, Baker, Bishop, Hammer, Furnstahl)

Summary

- EFT approach for the dilute Fermi system allows a straightforward computation of observables and reliable error estimates!

- We don't need to know the details of the short-distance behaviour of the underlying potential.
  $\rightarrow$ At low energy effective range parameters contain all necessary information.

- Many-body interactions enter the Lagrangian naturally.
  $\rightarrow$ no off-shell dependence of observables

- Nuclear matter cannot be described in this framework.
  $\rightarrow$ $k_F a_s > 1$ in nuclear matter!
  $\rightarrow$ pions are missing!

Outlook

- extend to finite temperature
- analyse Bose-Fermi mixtures
- include long-range interactions
- extend to large scattering length case $\rightarrow$ relevant for nuclear matter and atoms near Feshbach resonances
- integrate attractive interactions for Cooper pairing