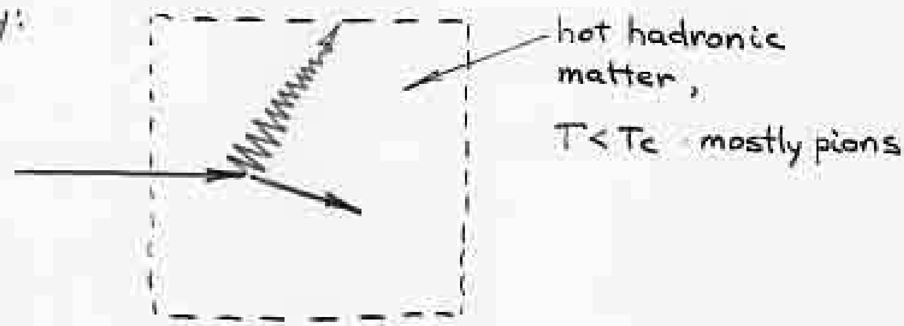
How QCD at finite T responds to time-dependent perturbations:

How do pions propagate in hot QCD?

Why:



Strictly: $T > 0$ - no particles

poles in real-time correlation functions $\rightarrow$ *quasiparticles*

Lattice: $0 < \tau < \frac{1}{T}$; $e^{-\omega\tau} \rightarrow e^{i\omega\tau}$ *problematic if $\omega \ll T$ ($t \gg 1/T$)*

or $\omega_0 = 0$, $\omega_1 = 2\pi T$, ...

Soft pions are special:

close to chiral limit

- Real-time propagation from static correlators — — measurable in Euclid (on lattice)
- $T \rightarrow T_c$: pion velocity $\rightarrow 0$ (phenomenology?)

D. Son, M.S.
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DEFINITIONS / RESULT

Real time $\langle \pi\pi \rangle$ correlator has poles at $\omega = \omega(\vec{q}) \leftarrow$ dispersion relation

$m_q = 0$: $\omega^2 = (u\vec{q})^2 + \dots$ u - pion velocity

$m_q \neq 0$: $\omega^2 = u^2(\vec{q}^2 + m^2) + \dots$ m - pion screening mass

$u \cdot m$ - pole mass

$u(T)$, $m(T)$ from static correlators :

m:
$$\int d\tau dV e^{-i\vec{q}\vec{x}} \frac{\langle \pi^a(x)\pi^b(0)\rangle}{\langle \bar\psi\psi\rangle^2} \overset{q,m\to 0}{=} \frac{1}{f^2} \frac{\delta^{ab}}{\vec{q}^2 + m^2} \quad ; \quad \pi^a = i\bar\psi\gamma_5\tau^a\psi$$

"decay const"

u: $\boxed{u^2 = \frac{f^2}{\chi_{IS}}}$

$$\int d\tau dV \langle A_0^a(x) A_0^b(0)\rangle = \delta^{ab}\chi_{IS} \quad ; \quad A_0^a = \bar\psi\gamma^0\gamma^5\frac{\tau^a}{2}\psi$$

axial isospin

$T=0$: $f^2 = \chi_{IS} = f_\pi^2$

$f(T)$, $\chi_{IS}(T)$ - from lattice ?

Derivation:

Lagrangian approach (conceptually incorrect beyond small q, m – no dissipation)

Microscopic (given) $\longleftrightarrow$ Effective (symmetries, momentum exp. $\to$ few parameters)

$$\mathcal{L}_{micro} = i\bar{\psi}\gamma^\mu D_\mu \psi - (\bar{\psi}_L M \psi_R + h.c.) + \mu_{I5} A_0^3$$

$M = diag(m_u, m_d)$; μ_{I5}: chem. potl. for A.I.

$\mathcal{L}_{eff}$: d.o.f. pions $\to \Sigma \in SU(2)$;

$$\mathcal{L}_{eff} = \frac{f_t^2}{4} Tr\, \partial_0 \Sigma \partial_0 \Sigma^+ - \frac{f_s^2}{4} Tr\, \partial_i \Sigma \partial_i \Sigma^+ + \frac{f_m^2}{2} Re\, Tr\, M\Sigma \quad (\mu_{I5} = 0)$$

$\downarrow$

$$u^2 = f_s^2 / f_t^2 \quad , \quad m^2 = m_q f_m^2 / f_s^2$$

$f_t, f_s, f_m \to$ by matching to microscopic theory derivatives w.r.t. μ_{I5} and $M(x)$

- How $\mathcal{L}_{eff}$ depends on μ_{I5}?

$$\partial_0 \Sigma \to \partial_0 \Sigma - \frac{i}{2} \mu_{I5} (\tau_3 \Sigma + \Sigma \tau_3)$$

Matching second derivative:

$$\chi_{I5} = \frac{\partial^2 \mathcal{L}_{eff}}{\partial \mu_{I5}^2} = f_t^2$$

- $\mathcal{L}_{eff}(M)$: $\frac{\partial \mathcal{L}}{\partial M}$: $f_m^2 = -\langle \bar{\psi}\psi \rangle$; $\frac{\partial^2 \mathcal{L}}{\partial M(x) \partial M(y)}$: $f_s = f$

$\downarrow$

$$f^2 m^2 = -m_q \langle \bar{\psi}\psi \rangle \quad \text{– GOR at finite } T$$

$$\underline{u^2 = f^2 / \chi_{I5}}$$

Perturbative check

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi^a\partial^\mu\phi^a + \frac{\mu^2}{2}\phi^a\phi^a - \frac{\lambda}{4}(\phi^a\phi^a)^2$$

$a = 1, 2, \ldots, N$

$\langle\phi^N\rangle = v_0 = \frac{\mu^2}{\lambda}$ – tree level

$\phi^N = v + \sigma$, $(\phi^1, \ldots, \phi^{N-1}) \equiv \vec{\pi}$

v: $\langle\sigma\rangle = 0$; $\lambda v^2 - \mu^2 + \frac{N+2}{12}\lambda T^2 = 0$ $\qquad T \sim v_0 \gg m_\sigma$

$\Rightarrow v^2 = v_0^2\left(1 - \frac{T^2}{T_c^2}\right)$, $T_c^2 = \frac{12}{N+2}\frac{\mu^2}{\lambda}$

u: $G_\pi^{-1}(q) = q_0^2 - \vec{q}^{\,2} - \Sigma(q)$, $\Sigma(q) =$

$$\Sigma(q) = -\frac{T^2}{3v^2}q_0^2 + O(\lambda^{1/2})\vec{q}^{\,2}$$

$$u^2 = \left(1 + \frac{T^2}{3v^2}\right)^{-1}$$

f: $f^2 = v^2$ $\qquad$ from definition: $\frac{G_\pi(q_0=0,\vec{q})}{v^2} = \frac{1}{f^2\vec{q}^{\,2}}$

χ: $\delta\mathcal{L}_\mu = \mu(\pi_1\partial_0\sigma - \sigma\partial_0\pi_1) + \frac{\mu^2}{2}\left[(v+\sigma)^2 + \pi_1^2\right]$

$$\chi = \frac{\partial\mathcal{L}_{1\,loop}}{\partial\mu^2} = v^2 + \frac{T^2}{3}$$

$$u^2 = \frac{f^2}{\chi}$$

Operator (hydrodynamic) approach

- *Effective description of soft, slow collective modes* $\equiv$ HYDRODYNAMICS

D.O.F.? (i) Conserved densities (T^{00}, ρ, etc)

(ii) Goldstone modes (π)

(iii) Near T_c : order parameters (σ)

To linear order: $\phi^a \equiv \frac{\pi^a}{\langle\bar\psi\psi\rangle}$ and A_0^a (parity odd)

Equations (expand in momenta, fields):

- $\partial_0 \phi^a = \frac{1}{\chi} A_0^a + \varkappa \nabla^2 \phi^a + \eta^a$; (local noise ← assumption) $(m_q = 0)$

- $\partial_0 A_0^a = -\partial_i A^{ai}$ and $A_i^a = -f^2 \partial_i \phi^a - D\partial_i A_0^a - \xi_i^a$;

$$\partial_0 A_0^a = f^2 \nabla^2 \phi^a + D\nabla^2 A_0^a + \partial_i \xi^{ai}.$$

Noise:

$$\langle \eta^a(x)\eta^b(0)\rangle = F_\eta \delta^{ab} \delta^4(x)$$

$$\langle \xi_i^a(x)\xi_j^b(0)\rangle = F_\xi \delta^{ab}\delta_{ij}\delta^4(x)$$

Fix parameters:

- canonical commut. $\langle[\phi^a, A_0^b]\rangle = i\delta^{ab}\delta^3(x) \Rightarrow \chi = \chi_{I5}$
- $\langle A_0 A_0\rangle$, $\langle\phi\phi\rangle$, $\langle A_0\phi\rangle$, etc can be computed and used to fix parameters.

  e.g. $F_\xi, F_\eta \sim T\varkappa D$ — fluctuation-dissipation

Poles: $q_0 = uq - i\Gamma_q/2$, with $u^2 = \frac{f^2}{\chi_{I5}}$

$$\Gamma_q = (D+\varkappa)q^2$$

$u\,(T \to T_c)$

$$u^2 = \frac{f^2}{\chi_{I5}}$$

$$\int d\tau\, dV\, e^{-i\vec{q}\vec{x}} \langle \pi^a(x)\,\pi^b(0)\rangle = \delta^{ab}\, \frac{\langle \bar{\psi}\psi\rangle^2}{f^2}\, \frac{1}{q^2} \quad ; \quad q \ll m_\sigma$$

Near T_c : $\quad m_\sigma \ll T$

for q: $\quad m_\sigma \ll q \ll T$ (scaling window)

$$\int d\tau\, dV\, e^{-i\vec{q}\vec{x}} \langle \pi^a(x)\,\pi^b(0)\rangle \sim \frac{1}{q^{2-\eta}}$$

To match at $q \sim m_\sigma$:

- $f^2 = A\, m_\sigma^{-\eta} \langle \bar{\psi}\psi\rangle^2$ $\quad$ η – universal

  $\Downarrow$ $\quad$ $m_\sigma \sim t^\nu$, $\langle \bar{\psi}\psi\rangle \sim t^\beta$ $\quad t = (T_c - T)/T_c$

  $f^2 \sim t^{2\beta - \nu\eta} = t^{(d-2)\nu} = t^\nu$ $\quad$ $\eta \simeq 0.03$, $\nu \simeq 0.73$, $\beta \simeq 0.38$: $O(4)$, $d=3$

  $f^2 \sim m_\sigma$

- χ_{I5} – finite at $T = T_c$.

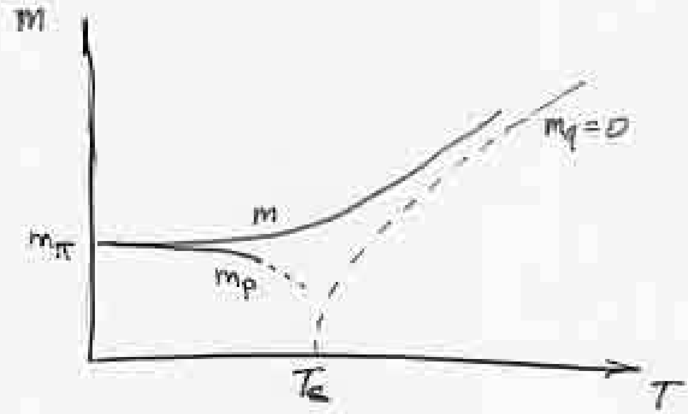
Thus: $\quad u^2 = \dfrac{f^2}{\chi_{I5}} \sim f^2 \sim t^\nu \to 0$ at T_c

$m(T) \quad T \to T_c$

$$m^2 = -\frac{m_q \langle \bar\psi \psi \rangle}{f^2} \sim m_q t^{\beta - \nu} \qquad \text{- grows}$$

Pole mass:

$$m_p^2 = u^2 m^2 = -\frac{m_q \langle \bar\psi \psi \rangle}{\chi_{15}} \sim m_q t^{\beta} \qquad \text{- drops}$$



- Phenomenological consequences (?)
  - Statistical models: $\exp(-\Delta m_p / T)$ enhancement of pion abundance
  - $u < 1$: Cherenkov pions?