Dynamical Polarisabilities:
A New Old Tool to Study Nucleons

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Aim: Which internal degrees of freedom rule the nucleon at low energies?

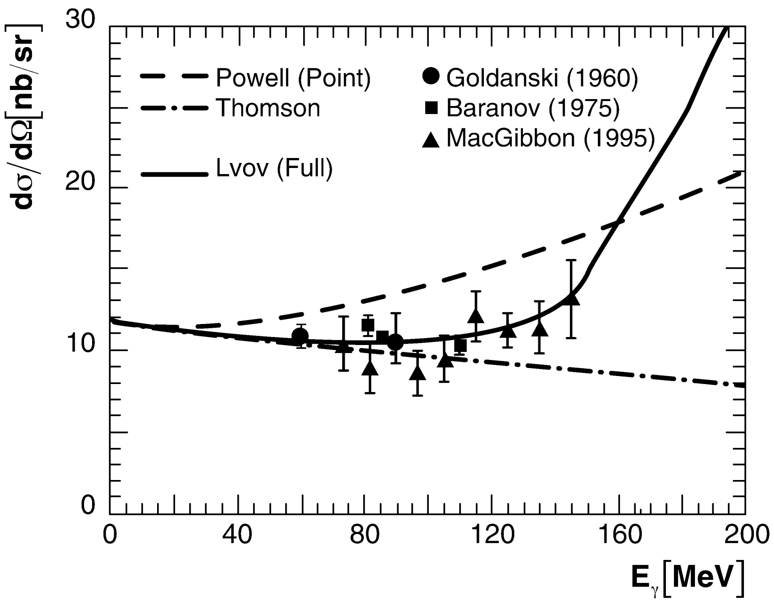
hg/T. R. Hemmert: nucl-th/0110006;
hg/T. R. Hemmert/R. Hildebrandt (diploma student)/B. Pasquini: in preparation.

What are Polarisabilities?

(a) Setting the Stage: Compton Scattering off the Proton

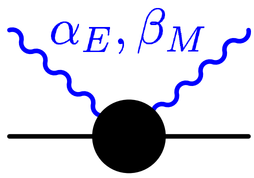
Even at low photon energy ω, nucleon is more than a point-like spin $\frac{1}{2}$ object with anomalous magnetic moment κ:

Compton scattering $\gamma N \to \gamma N$ ill described by Powell cross section.



$\Longrightarrow$ Photon resolves internal target structure by effective interaction; dominating term:

$$\mathcal{L}_{\text{pol}} = 2\pi \left[\bar{\alpha}_E \, \vec{E}^2 + \bar{\beta}_M \, \vec{B}^2\right] N^\dagger N$$



(c) More General: Dynamical Polarisabilities

Generalisation of structure Lagrangean:

Most general gauge, Lorentz, iso-spin, ... invariant point-like interaction of nucleon with electro-magnetic field of a real photon of definite multipolarity and non-zero energy ω, e.g. for electric: (L'vov/Petrunkin)

$$\mathcal{L}_{\text{pol},\,E} = 4\pi \left[\frac{1}{2}\,\bar{\alpha}_E\,\vec{E}^2 \;+\; \frac{1}{2}\,\bar{\alpha}_{E\nu}\,\dot{\vec{E}}^2 \;+\; \frac{1}{12}\,\bar{\alpha}_{E2}\,E_{ij}^2 \;+\; \ldots \right] N^\dagger N$$

static dipole — dipole dispersion — quadrupole

$$E_{ij} := \tfrac{1}{2}(\partial_i E_j + \partial_j E_i)$$

$$\Longrightarrow \bar{\alpha}_E \;+\; \bar{\alpha}_{E\nu}\,\omega^2 + \;\ldots\; \to\; \alpha_{E1}(\omega)$$

Coefficients become energy dependent: Dynamical (Multipole) Polarisabilities.

$$\mathcal{L}_{\text{pol},\,E} = 4\pi \left[\frac{1}{2}\,\alpha_{E1}(\omega)\,\vec{E}^2 \;+\; \frac{1}{12}\,\alpha_{E2}(\omega)\,E_{ij}^2 \;+\; \ldots \right] N^\dagger N$$

Contain information about temporal response/dispersive effects of internal nucleon degrees of freedom.

$$\mathcal{L}_{\text{pol}} = 4\pi \left\{ \frac{1}{2} \left[\alpha_{E1}(\omega)\,\vec{E}^2 \;+\; \beta_{M1}(\omega)\,\vec{B}^2 \right] + \frac{1}{12} \left[\alpha_{E2}(\omega)\,E_{ij}^2 \;+\; \beta_{M2}(\omega)\,B_{ij}^2 \right] \right\} N^\dagger N \;+\; \ldots$$

Static limit: $\alpha_{E1}(\omega = 0) = \bar{\alpha}_E$

Extracting Dynamical Polarisabilities

hg/T. R. Hemmert: nucl-th/0110006

Equivalent definition from spin-independent Compton scattering amplitudes

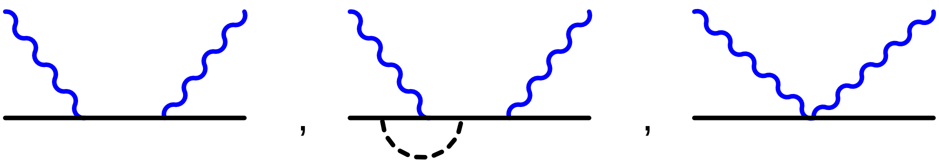
$$T(\omega,\cos\theta) = A_1(\omega,\cos\theta)\ \left(\vec{\epsilon}'^{\,*}\cdot\vec{\epsilon}\right) + A_2(\omega,\cos\theta)\ \left(\vec{\epsilon}'^{\,*}\cdot\hat{\vec{k}}\right)\ \left(\vec{\epsilon}\cdot\hat{\vec{k}}'\right) + \dots$$

(i) Choose frame of reference: centre of mass; $\theta = \measuredangle(\vec{k},\vec{k}')$.

(ii) Subtract "nucleon pole" terms following L'vov,

i.e. two photon interaction with point-like nucleon (Powell).

$$\bar{A}_i(\omega,\cos\theta) = A_i(\omega,\cos\theta) - A_i^{\text{pole}}(\omega,\cos\theta)$$

e.g. 

(iii) Expand remainder of amplitudes in polarisabilities, identified via decomposition of invariant amplitudes $R_i(\omega, z)$ into multipole amplitudes $f^{ll'}_{TT'}(\omega)$ of transitions $Tl \to T'l'$, $T = E, M$, after Ritus 1957/Contogouris 1962/Nagashina 1965/Babusci et al. 1998.

$W = \omega + \sqrt{M^2 + \omega^2}$: cm energy, $z = \cos\theta$

$$\bar{A}_1(\omega, z) = \frac{4\pi W}{M}\Big[\Big(\alpha_{E1}(\omega) + z\beta_{M1}(\omega)\Big)\omega^2 + \\ + \frac{1}{12}\Big(z\alpha_{E2}(\omega) + (2z^2-1)\beta_{M2}(\omega)\Big)\omega^4 + \dots\Big]$$

$$\bar{A}_2(\omega, z) = -\frac{4\pi W}{M}\Big[\beta_{M1}(\omega)\,\omega^2 + \\ + \frac{1}{12}\Big(-\alpha_{E2}(\omega) + 2z\beta_{M2}(\omega)\Big)\omega^4 + \dots\Big]$$

(iv) Project out lowest polarisabilities using Legendre polynomials:

$$\alpha_{E1}(\omega) = \frac{3M}{32\pi W\omega^2}\int_{-1}^{1} \mathrm{d}z\,\Big[(1+z^2)\,\bar{A}_1(\omega,z) + z(z^2-1)\,\bar{A}_2(\omega,z)\Big]$$

$$\beta_{M1}(\omega) = \frac{3M}{32\pi W\omega^2}\int_{-1}^{1} \mathrm{d}z\,\Big[2z\,\bar{A}_1(\omega,z) + (z^2-1)\,\bar{A}_2(\omega,z)\Big]$$

$$\alpha_{E2}(\omega) = \frac{15M}{8\pi W\omega^4}\int_{-1}^{1} \mathrm{d}z\,\Big[z^3\,\bar{A}_1(\omega,z) + (2z^4-3z^2+1)\,\bar{A}_2(\omega,z)\Big]$$

$$\beta_{M2}(\omega) = \frac{15M}{8\pi W\omega^4}\int_{-1}^{1} \mathrm{d}z\,\Big[(3z^2-1)\,\bar{A}_1(\omega,z) + \frac{4}{5}z(z^2-1)\,\bar{A}_2(\omega,z)\Big]$$

Polarisability Mechanisms & Non-Zero Energy Effects

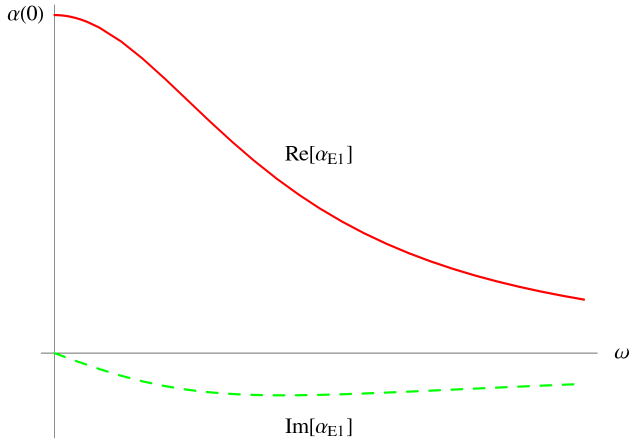
Dynamical polarisabilities: Response of internal degrees of freedom to external, real photon field of definite multipolarity and non-zero energy.

(1) Orientation Relaxation

External field lines up

originally randomly oriented,

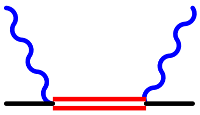
non-zero static multipole moment.

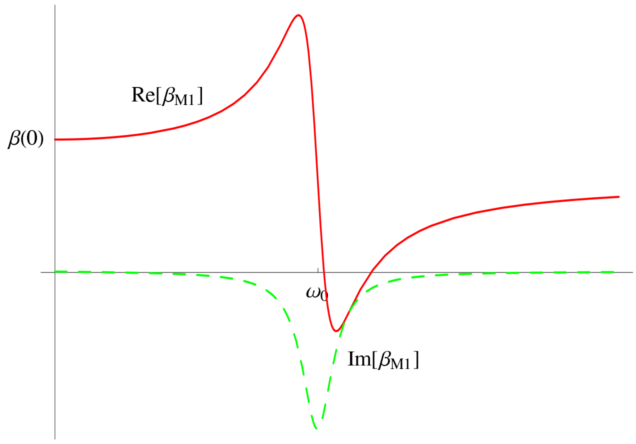


(2) Resonance Effects: Displacement Polarisation

Displacement of charged particles with reset force induces multipole moment; characteristic resonance.

In nucleon e.g. $\Delta(1232)$:

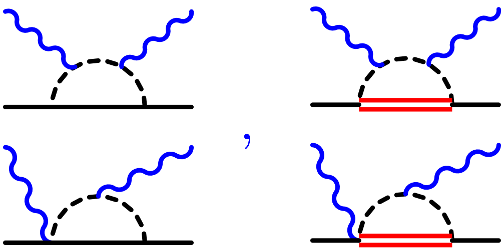




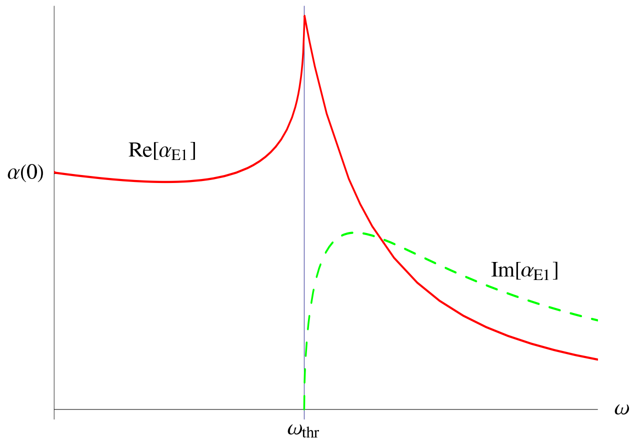
(3) Particle Production Threshold

Cusp at ω_{thr}.

Lowest nucleonic thresholds: $N\pi$, $\Delta\pi$.



etc.



Competing Effects in the Nucleon

Most prominent:

Why is static $\bar{\beta}_M^{(p)} \approx 1.5 \times 10^{-4}\ \ \mathrm{fm}^3 \approx \dfrac{1}{10}\ \bar{\alpha}_E$ so small in the proton?

Dis-entangling the magnetic scalar dipole contributions:

– Large para-magnetic (spin flip) term from strong $N \to \Delta$ magnetic dipole transition: (e.g. Mukhopadhyay/Nathan/Zhang 1993)

$$\delta\bar{\beta}_{M,\Delta}^{(p)} \approx +7 \times 10^{-4}\ \ \mathrm{fm}^3$$

– HBχPT hypothesis: (Bernard/Kaiser/Meißner/Schmidt 1993)

Large dia-magnetic induced dipole moment with large $\ln m_\pi$ dependence from pionic currents on nucleon surface in proton:

$$\delta\bar{\beta}_{M,\pi}^{(p)} \approx -5 \times 10^{-4}\ \ \mathrm{fm}^3$$

$\Longrightarrow \bar{\beta}_M^{(p)} = \delta\bar{\beta}_{M,\pi}^{(p)} + \delta\bar{\beta}_{M,\Delta}^{(p)}$ “fine tuned”

Different ω dependence expected: scales, mechanisms.

$\Longrightarrow$ Likely to be dis-entangled by dynamical polarisabilities.

What happens in the neutron?

$\Longrightarrow$ Deuteron Compton scattering

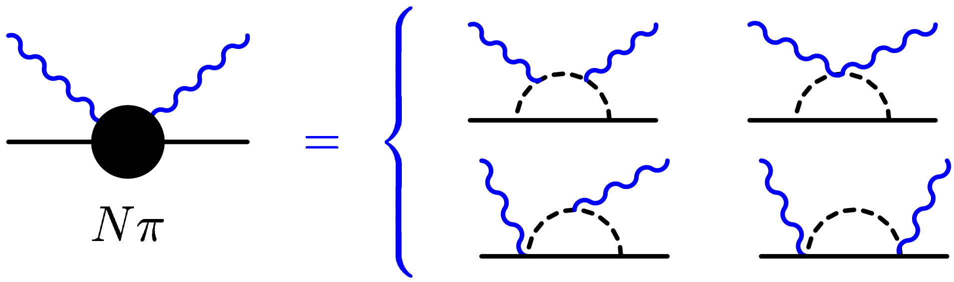
Dynamical Polarisabilities: Theory and Phenomenology

(a) Leading Order Heavy Baryon Chiral Perturbation Theory

hg/T. R. Hemmert: nucl-th/0110006, hg/T. R. H./R. Hildebrandt/B. Pasquini: in preparation.

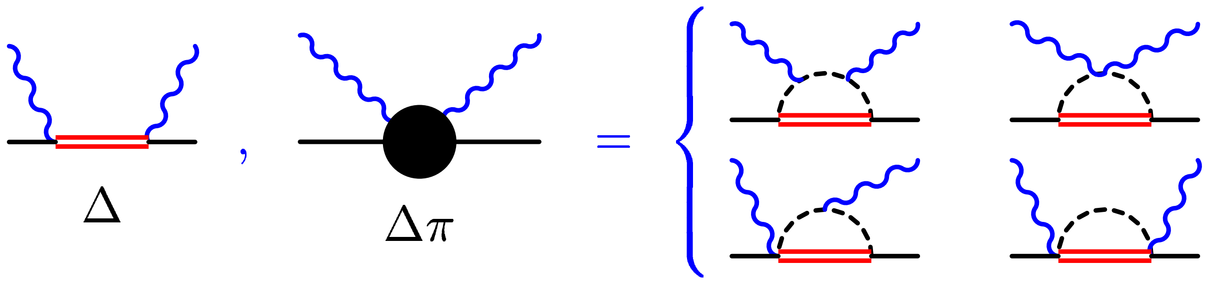
Microscopic explanation by dominant low energy processes.

– LO “classical” HBχPT: Point-like nucleon surrounded by pion cloud.
(amplitudes: Bernard/Kaiser/Kambor/Meißner 1992).



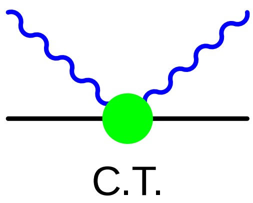
$\Longrightarrow$ Expect cusp at $\omega \approx m_\pi$ reflecting one pion production threshold.

+ LO Small Scale Expansion: Includes Δ and $\Delta\pi$ graphs.
(Hemmert/Holstein/Kambor 1995;
amplitudes: Hemmert/Holstein/Kambor/Knöchlein 1998)



Δ without width: Expect qualitative agreement above $\omega \approx 200\ \mathrm{MeV}$.

+ LO Modified SSE (Hemmert 2001): Includes counter terms for α_{E1}, β_{M1}.



Naïvely $\sim \dfrac{e^2}{2\pi}\,\dfrac{1}{M^3} \approx 1.5 \times 10^{-4}\ \mathrm{fm}^3$, i.e. NLO,

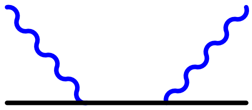
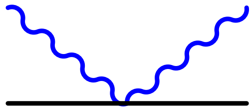
but turn out anomalously large: $\delta(\bar{\alpha};\bar{\beta}) \approx -5; -12$!

Strengths fixed by static polarisabilities.

$\Longrightarrow$ Energy dependence predicted.

– At LO, pole contributions (Powell) easily subtracted:

non-relativistically: $A_1^{\text{pole}}(\omega) = -\dfrac{e^2}{M}$, $A_2^{\text{pole}}(\omega) = \dfrac{e^2\omega}{M^2}$

from  , 

Note on Threshold Corrections

LO χDyn.: N, Δ static $\Longrightarrow$ “Lab. $=$ cm $=$ Breit”, $\omega_\pi = m_\pi$

Correction of one pion cut position in cm to

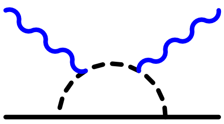
$$\omega_\pi = \frac{m_\pi (m_\pi + 2M)}{2(m_\pi + M)} \approx 131 \ \ \mathrm{MeV}$$

formally only resummation of higher order terms,

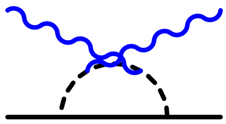
but known correction $\Longrightarrow$ better convergence.

$\Longrightarrow$ Re-summation such that

all physical one pion production cuts open at ω_π, e.g.



crossing symmetry still holds when comparing to e.g.



$\Longrightarrow$ low energy theorems conserved

$\Longrightarrow$ $N\pi$ contributions to static polarisabilities:

$$\alpha_{E1,N\pi}(\omega = 0) \quad = \quad \frac{5\, e^2\, g_A^2}{24\, (4\pi F_\pi)^2\, m_\pi} \qquad \text{unchanged}$$

$$\beta_{M1,N\pi}(\omega = 0) \quad = \quad \frac{e^2\, g_A^2}{48\, (4\pi F_\pi)^2\, m_\pi} \qquad \text{unchanged}$$

$$\alpha_{E2,N\pi}(\omega = 0) \quad = \quad \frac{7e^2\, g_A^2}{40 (4\pi f_\pi)^2\, m_\pi^3} \underbrace{\left[1 + \frac{2185 m_\pi^2}{294 M^2} + \frac{3 m_\pi}{7\pi M} \right]}_{\approx 1.2}$$

$$\beta_{M2,N\pi}(\omega = 0) \quad = \quad - \frac{3e^2\, g_A^2}{40 (4\pi f_\pi)^2\, m_\pi^3} \overbrace{\left[1 - \frac{305 m_\pi^2}{18 M^2} + \frac{37 m_\pi}{3\pi M} \right]}^{\approx 1.2}$$

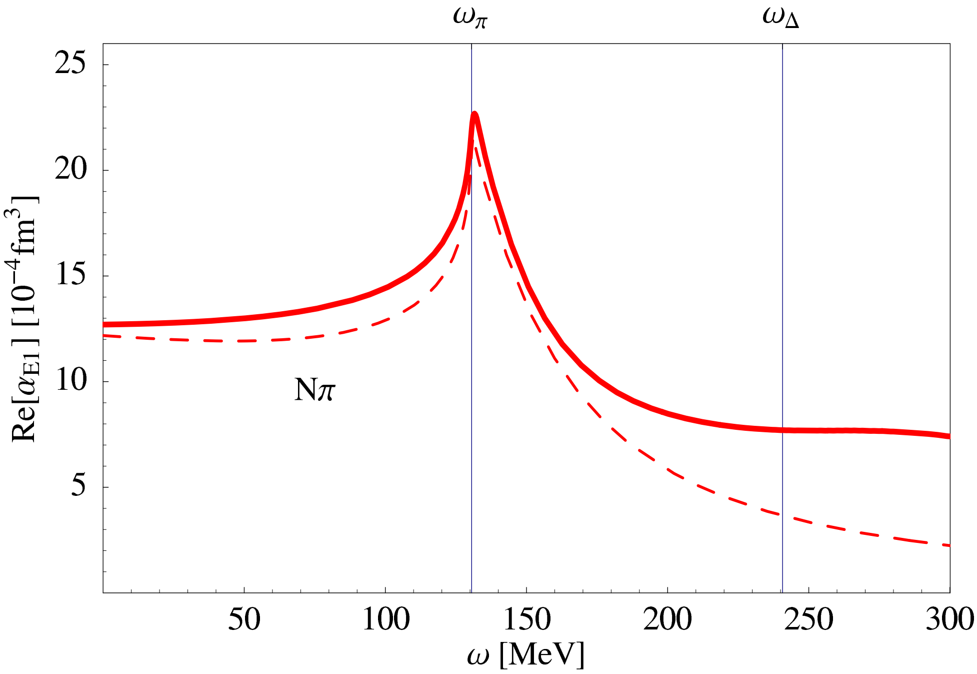
(b) Phenomenology: Dispersion Relations

hg/Hemmert/Hildebrandt/Pasquini: in preparation.

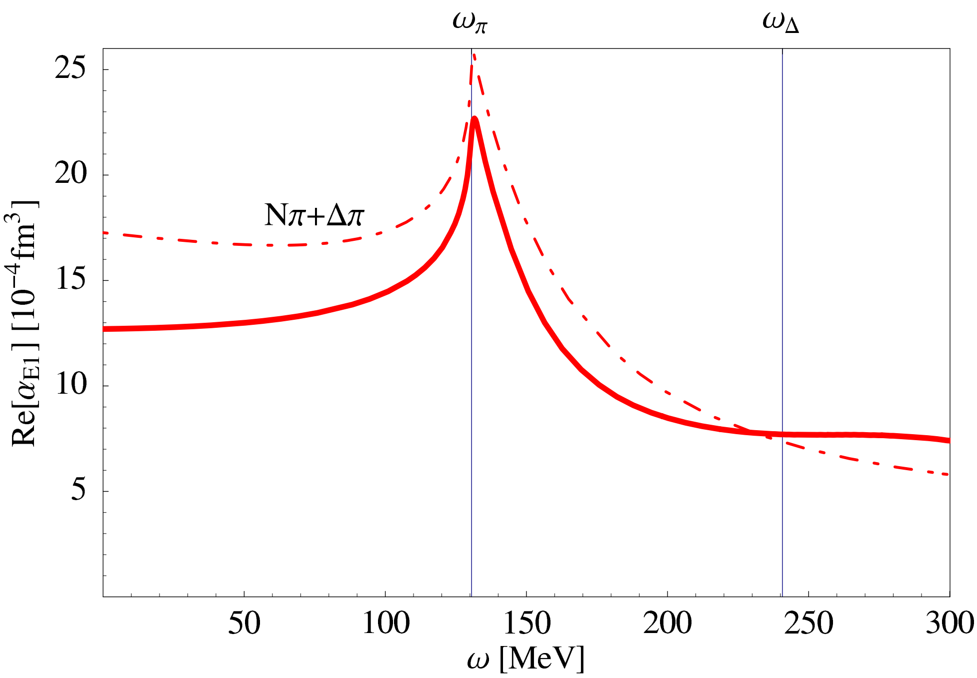
Extraction of polarisabilities from data by minimal theoretical prejudice, no microscopic explanation attempted.

- Static dipole polarisabilities fit to experiment.
- Static quadrupole polarisabilities from subtracted dispersion relations.
- Energy dependent effects subsumed into integral over photo-absorption cross section $\gamma N \to X$.
  $\Longrightarrow$ Experimental input from different régime.
- High energy behaviour and data parameterisation: SAID2002.
- Major source of error: insufficient neutron data.

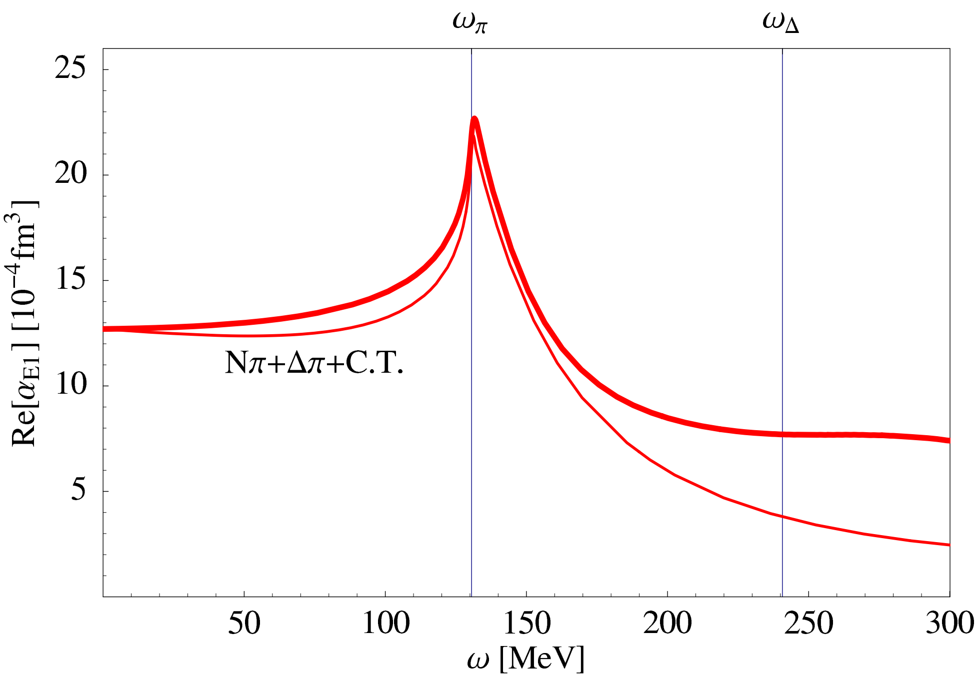
$\alpha_{E1}(\omega)$: Pion cusp clearly visible, but hardly dispersive at $\omega < 100$ MeV.



$\Delta\pi$ continuum



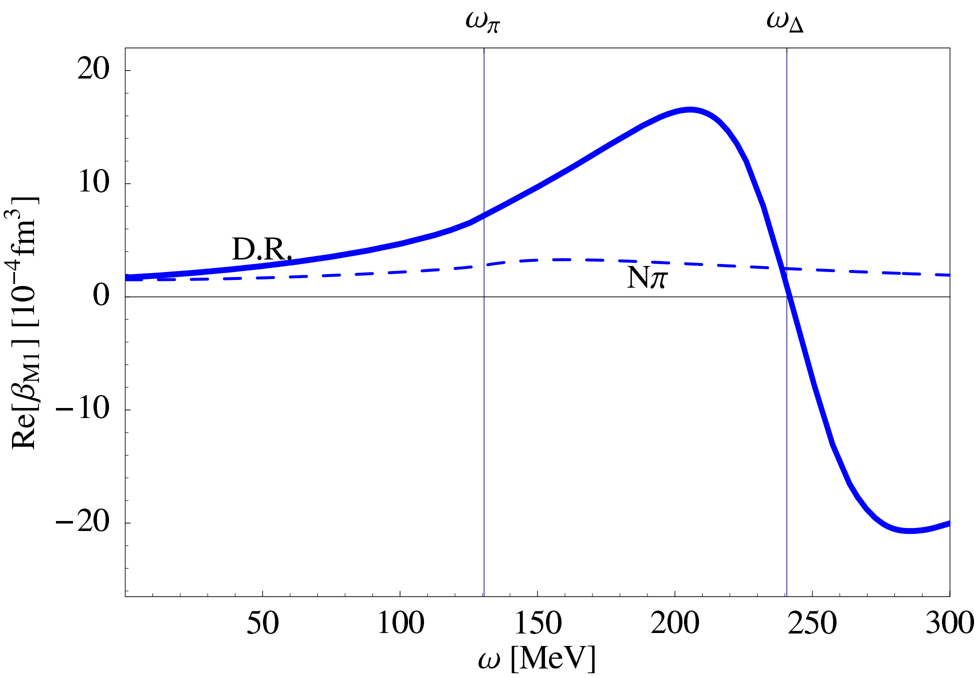
nearly cancels C.T. $\delta\alpha_{E1} \approx -5.2$ in LO MSSE.

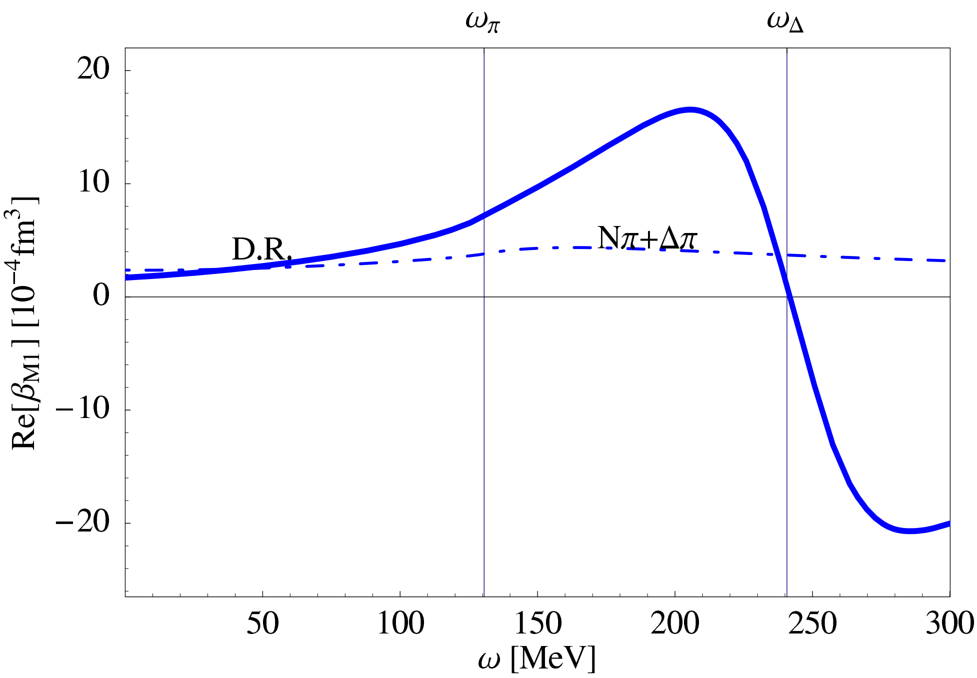


Naïvely $\left|\delta\alpha_{E1}\right| \sim \frac{e^2}{2\pi}\,\frac{1}{M^3} \approx 1.5 \times 10^{-4}\ \ \mathrm{fm}^3$, i.e. NLO.

But now quite large. . .

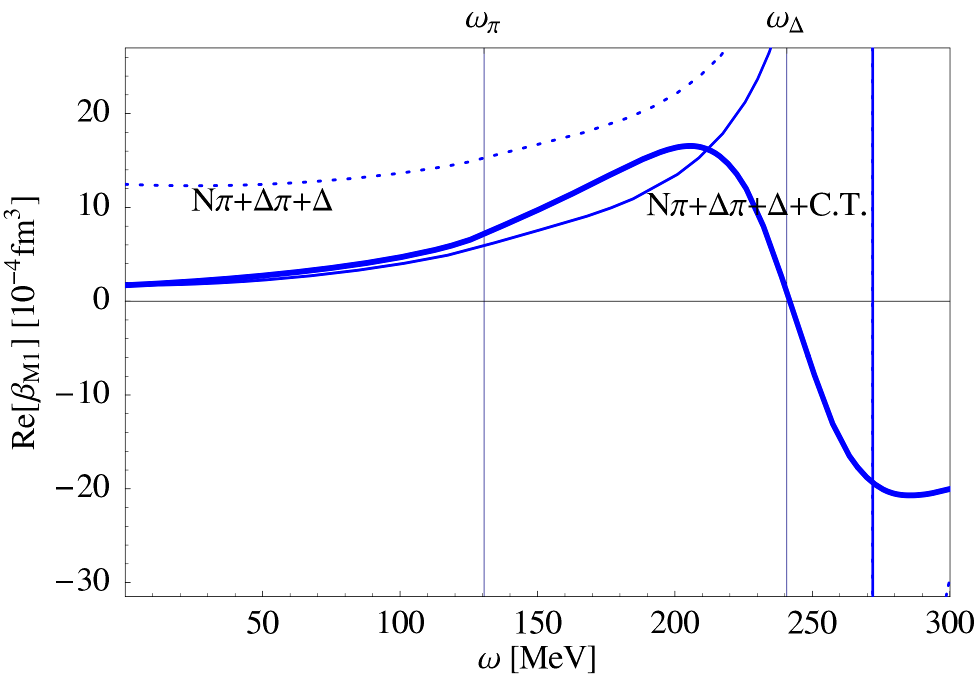
$\beta_{M1}(\omega)$: Strong ω dependence; Δ in D.R.: para-magnetic $M1 \to M1$.



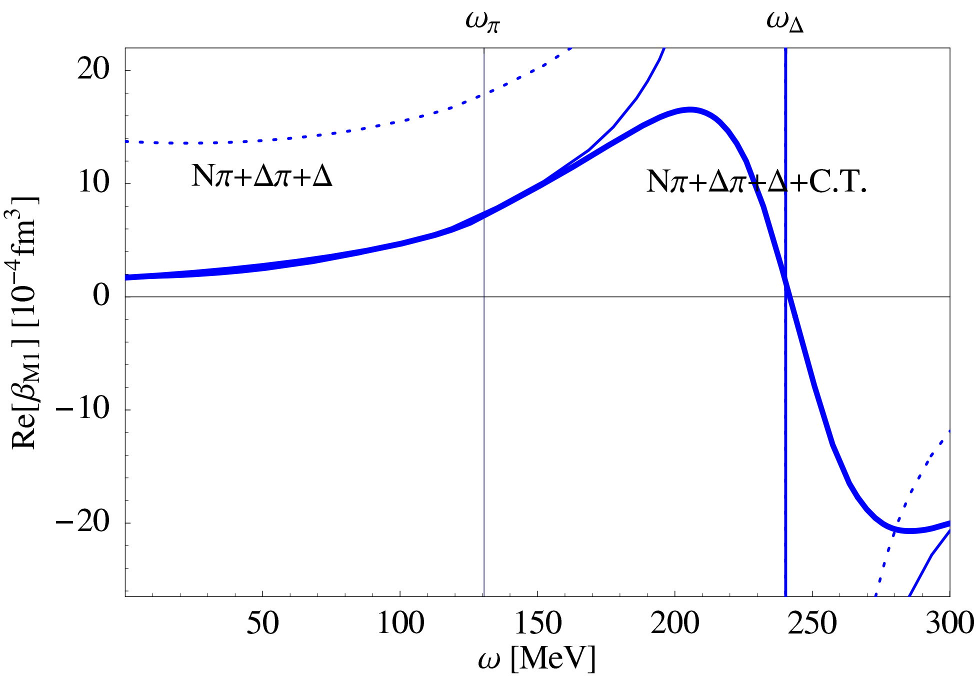


ω_π
ω_Δ
20
10
0
−10
−20
$\mathrm{Re}[\beta_{M1}]\ [10^{-4}\mathrm{fm}^3]$
D.R.
$N\pi+\Delta\pi$
50
100
150
200
250
300
ω [MeV]

Δ in LO (M)SSE without width; C.T. $\delta\beta_{M1} \approx -11$ needed.



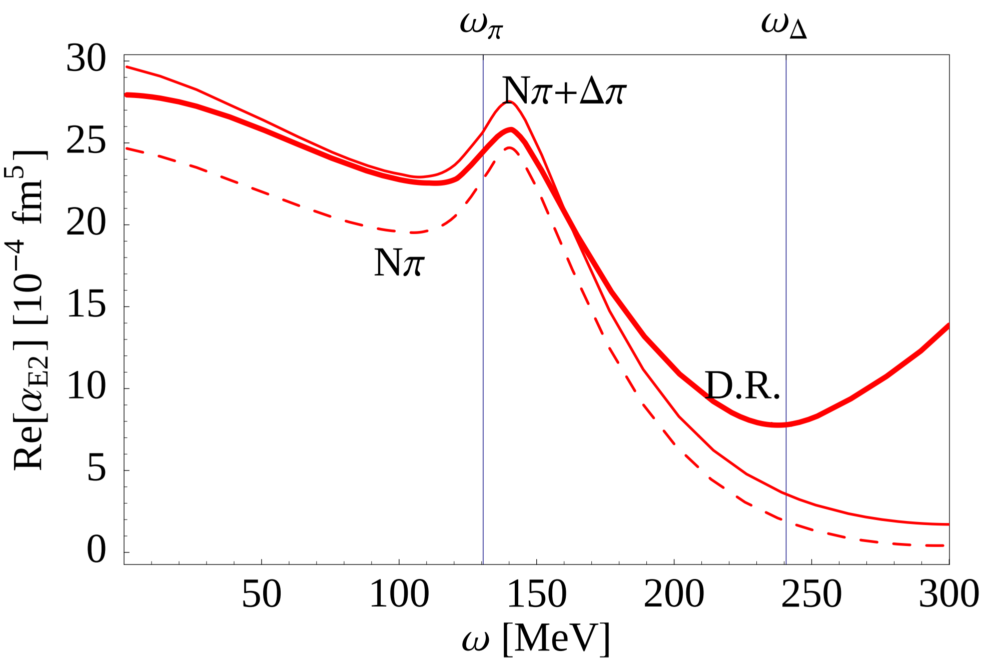
Δ in LO (M)SSE without width; C.T. $\delta\beta_{M1} \approx -11$ needed.



$\alpha_{E2}(\omega)$: Δ in D.R.: No $E2 \to E2$ of Δ visible.

Relaxation at low ω; cusp as $\omega \to m_\pi$ prevents sharp drop.

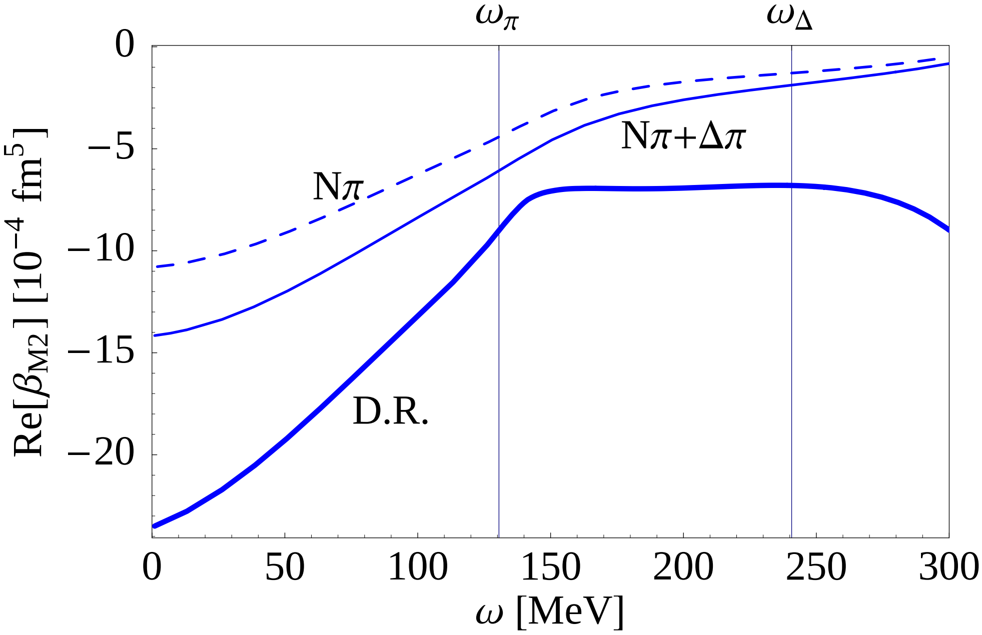
Well reproduced in LO (M)SSE (no C.T.).



$\beta_{M2}(\omega)$: Strong dispersion even at low ω not by $N\pi/\Delta\pi$ dia-magnetisms.

Hypothesis: Strong dia-magnetic quadrupole relaxation?

C.T. would still yield wrong slope.

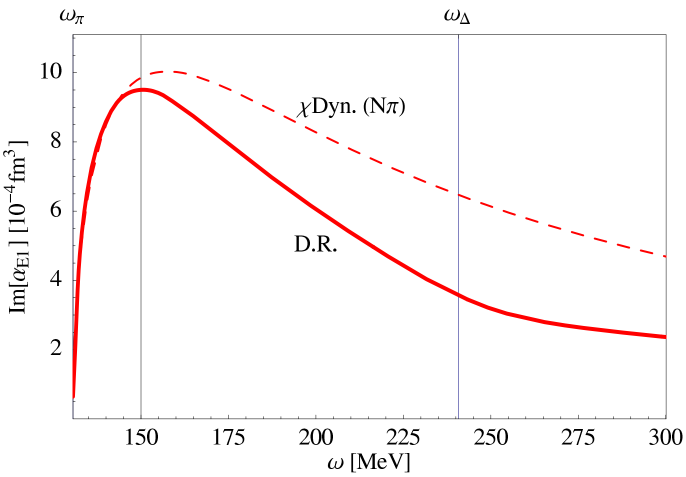


(d) Imaginary Parts of Iso-Scalar Polarisabilities

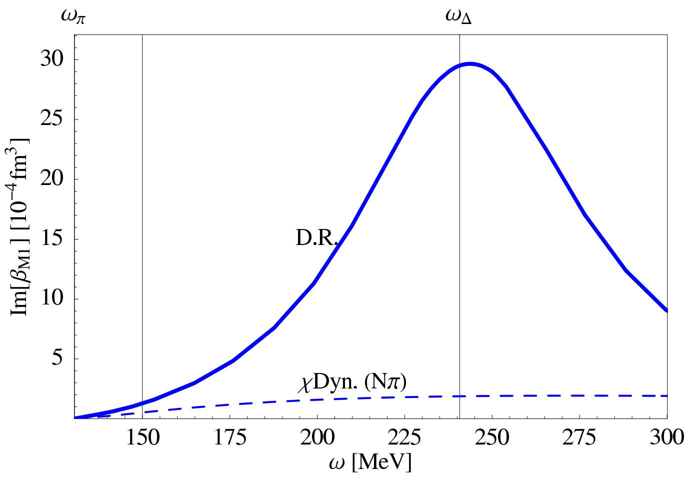
LO MSSE: Δ without width; only pion production threshold ($N\pi$ graphs).

Non-zero Δ width clearly seen in DR.

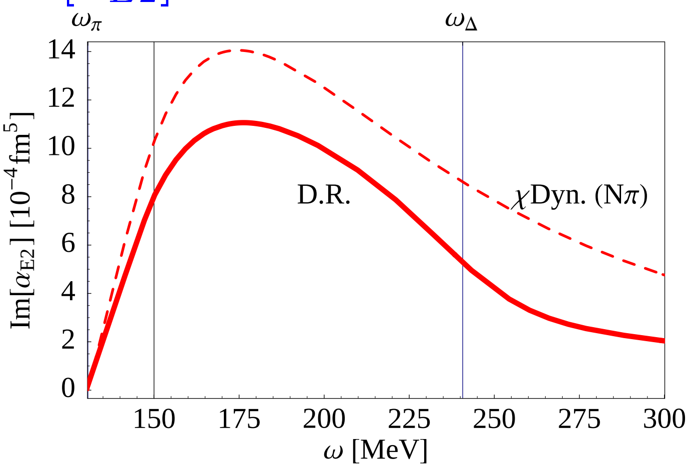
$\mathrm{Im}[\alpha_{E1}]$



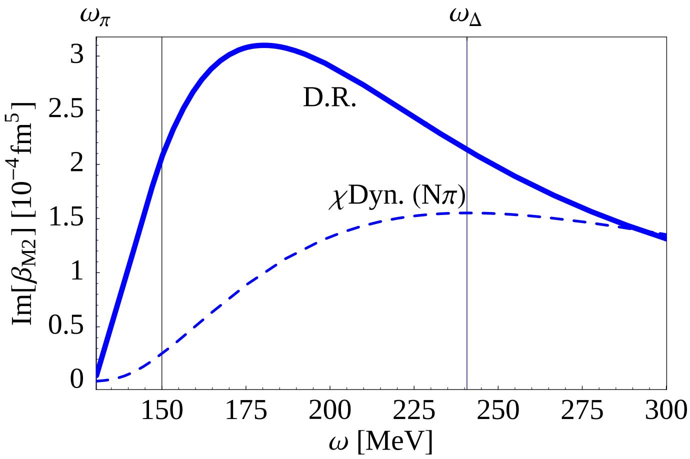
$\mathrm{Im}[\beta_{M1}]$



$\mathrm{Im}[\alpha_{E2}]$



$\mathrm{Im}[\beta_{M2}]$

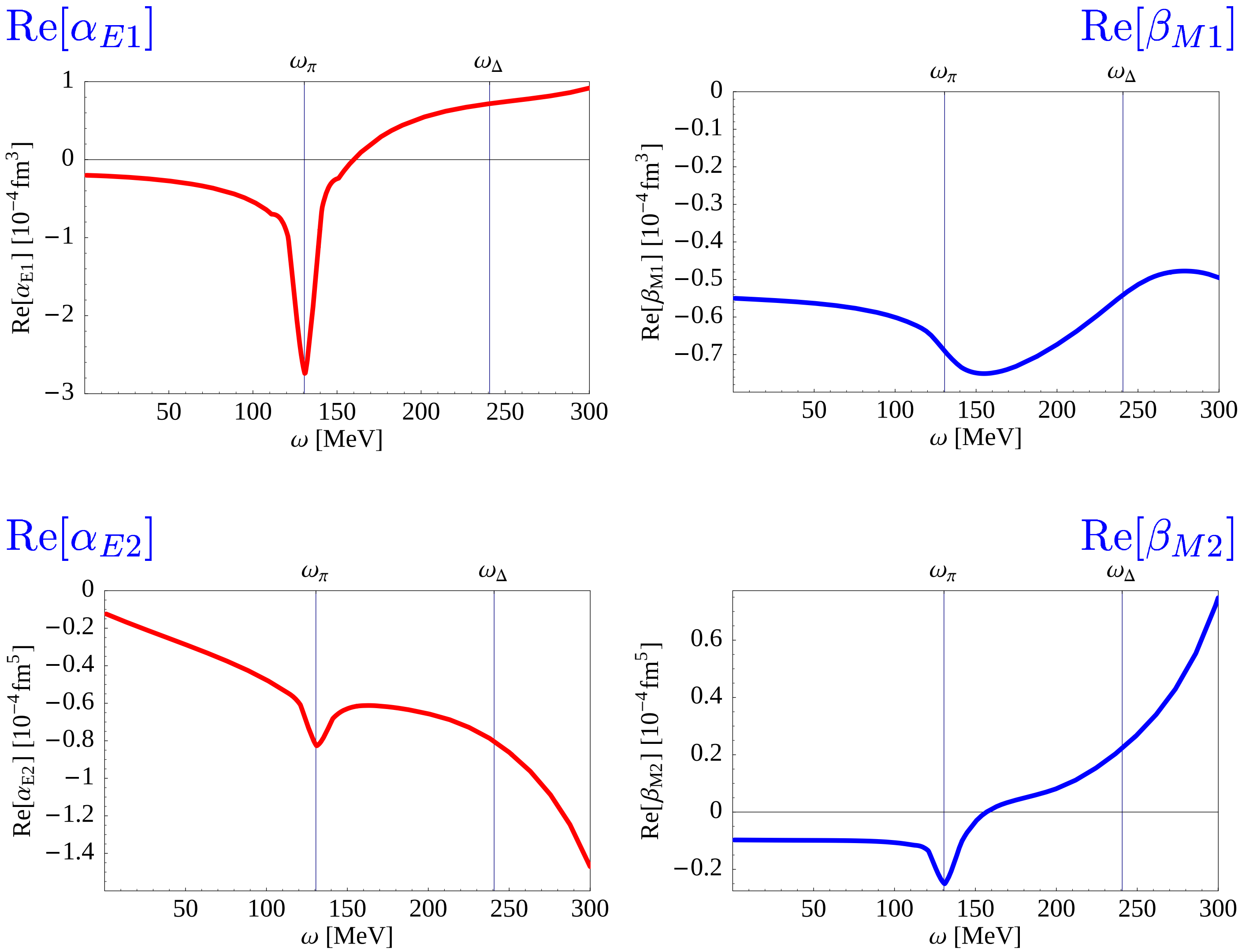


(e) Iso-Vector Polarisabilities

$\frac{1}{10}$th of iso-scalar polarisabilities in DR.

Zero in χDyn. at LO.

Probes accurracy of data set of Dispersion Relations: Limits by neutron data.



(f) Iso-Scalar Spin-Dependent Dynamical Polarisabilities

Decomposition of invariant amplitudes
into multipole amplitudes $f^{ll'}_{TT'}(\omega)$ crucial for consistent definition.

Example:

$$\gamma_{E1E1}(\omega) = \frac{3M}{64\pi\, W\, \omega^3} \int\limits_{-1}^{1} \mathrm{d}z \Big[(z^2-3)\ \bar{A}_3(\omega,z) + \\ +4z(z^2-1)\ [\bar{A}_4(\omega,z) + 2\bar{A}_5(\omega,z)] + 4(z^2-1)\ \bar{A}_6(\omega,z) \Big]$$

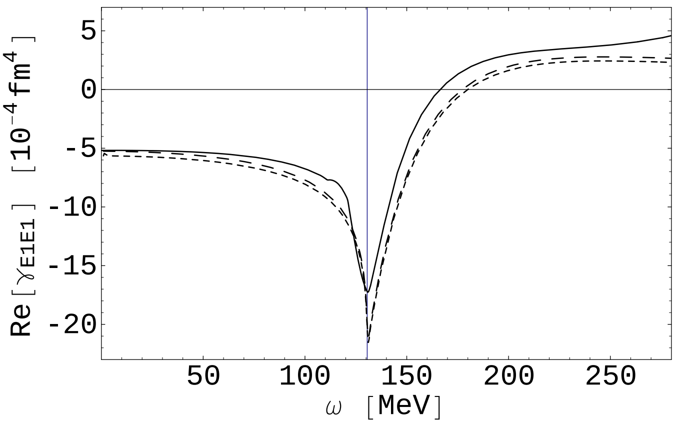
– Pure polarisabilities $\gamma_{E1E1}, \gamma_{M1M1}, \gamma_{E2E2}, \gamma_{M2M2}$ quite good;

– Mixed polarisabilities $\gamma_{E1M2}, \gamma_{M1E2}$ difficult: small;

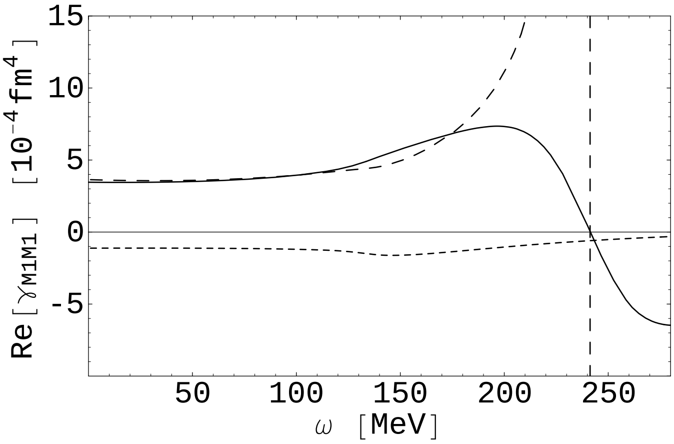
– Even spin physics is dominated by $N\pi$ (dotted), except manifest Δ;

– Δ in γ_{M1E2} seen in D.R.(solid) $\Longrightarrow$ How in χDyn (dashed)?

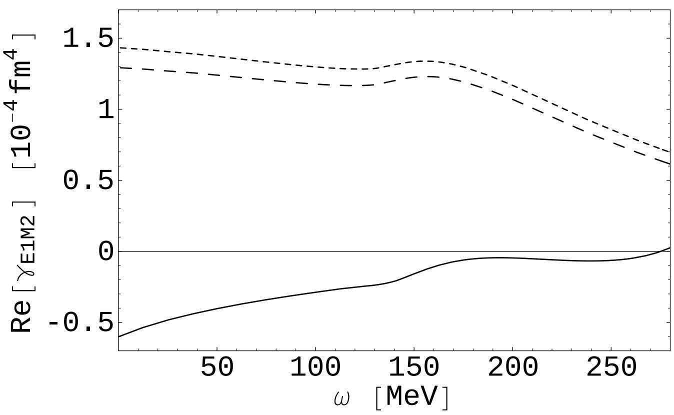
$\mathrm{Re}[\gamma_{E1E1}]$



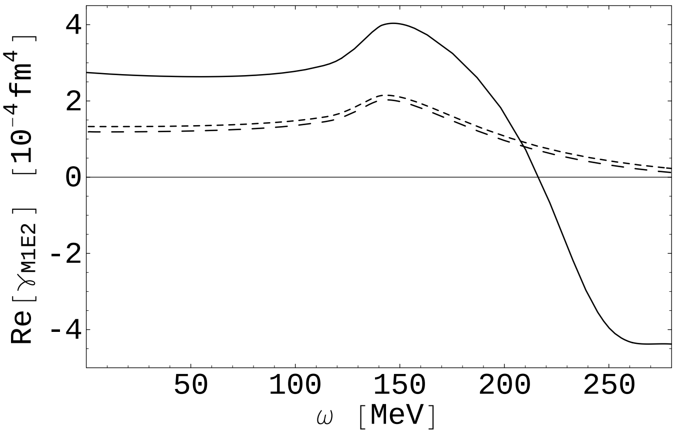
$\mathrm{Re}[\gamma_{M1M1}]$



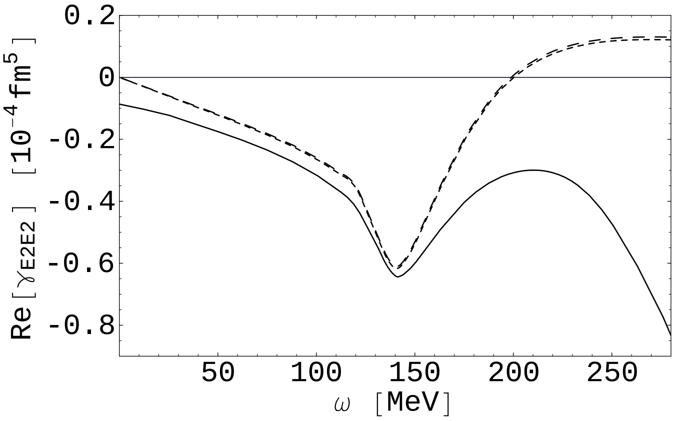
$\mathrm{Re}[\gamma_{E1M2}]$



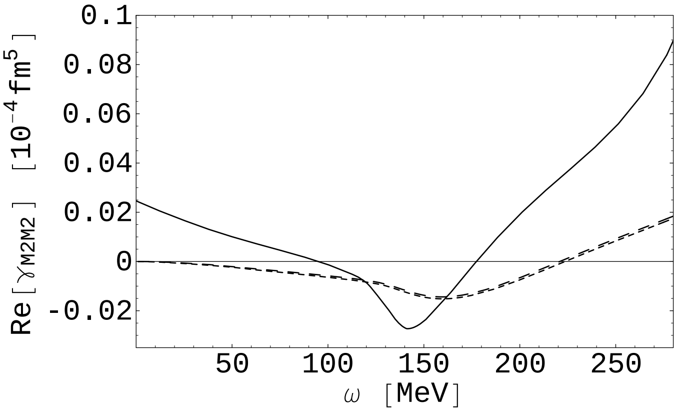
$\mathrm{Re}[\gamma_{M1E2}]$



$\mathrm{Re}[\gamma_{E2E2}]$



$\mathrm{Re}[\gamma_{M2M2}]$



From Polarisabilities to Compton Scattering off Proton

– Dynamical polarisabilities contain neither more nor less information than Compton amplitudes, but more readily accessible.

– Dispersion Relations take completely different data ("high" energy $\gamma N \to X$).

$\Longrightarrow$ Add Born terms,
insert first few dyn. pols.,
compare to Saskatoon (Hallin et al. 1995, higher ω, cm frame)
and MAMI (Olmos de Leon 2001, lower ω, Lab frame) data.

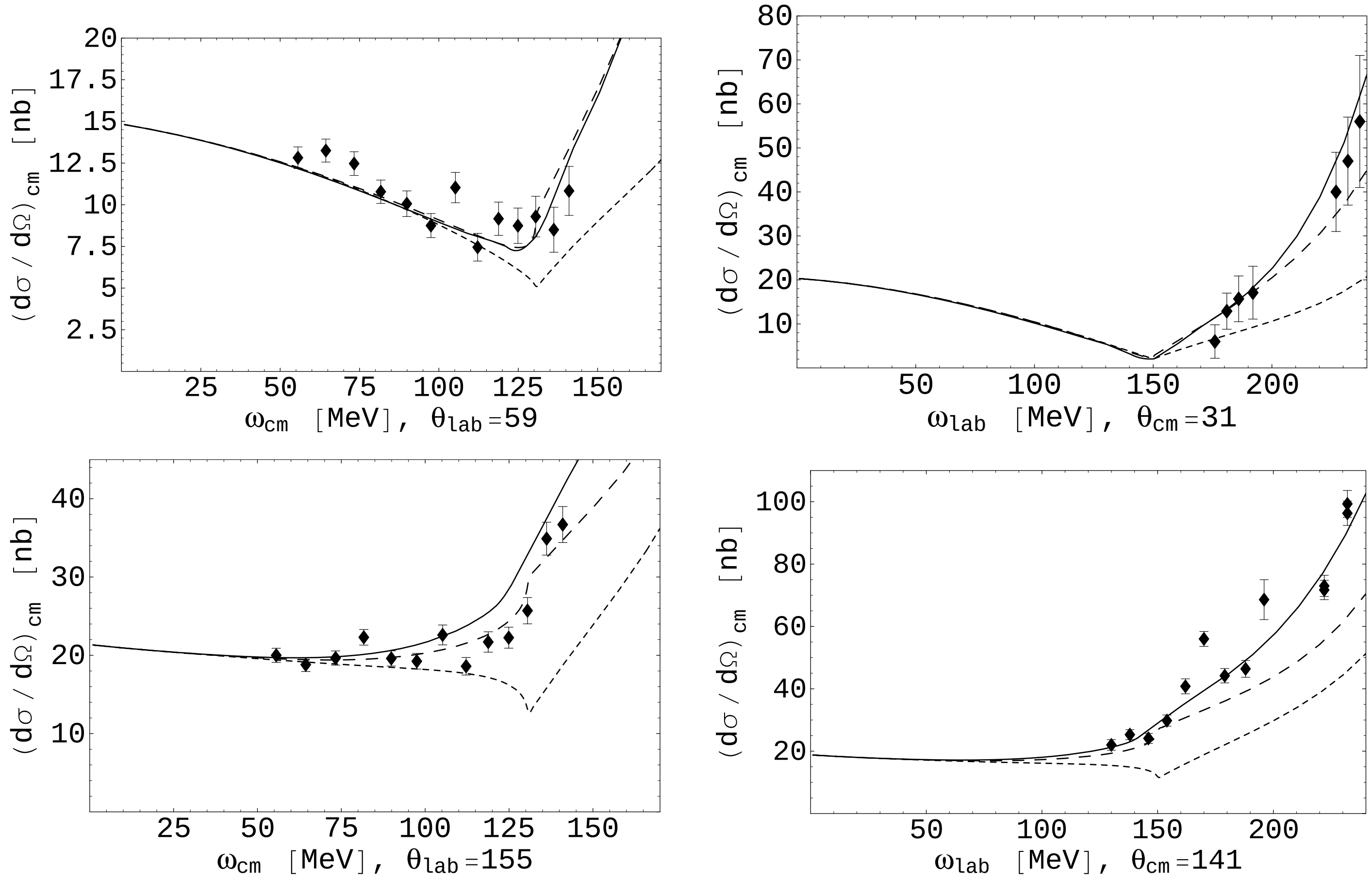
– Solid: D.R.; dashed: χDyn dipole and quadrupole;

– No dipole and quadrupole spin polarisabilities gives large effect (dashed);

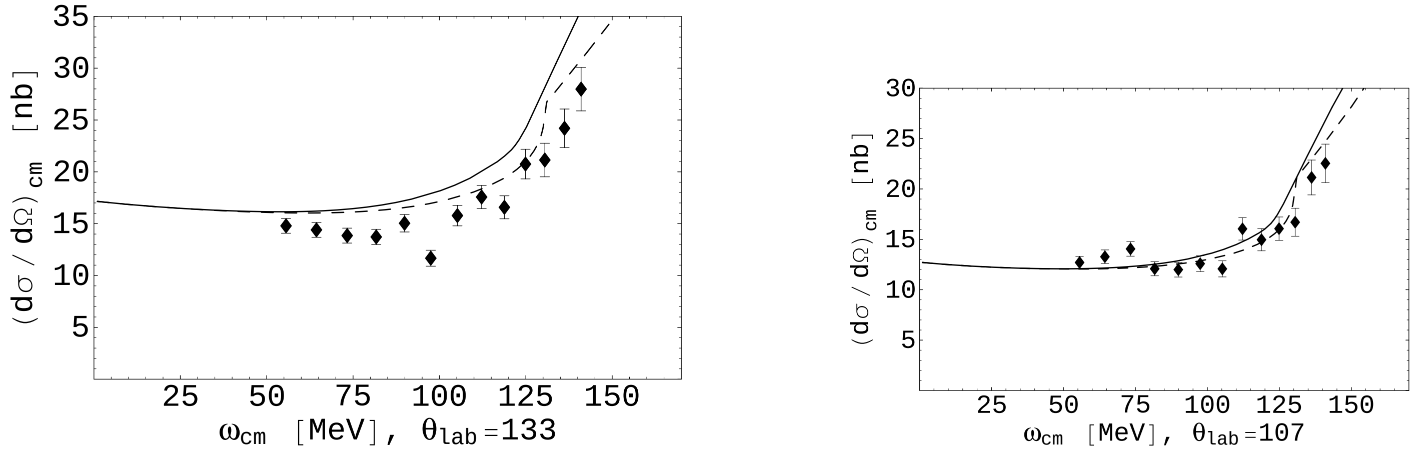
– Neglecting Δ width gives large effect (large ω, ϑ);

– Taking only dipole polarisabilities gives tiny effect.

MAMI Saskatoon



One MAMI dataset seems not to fit into the picture. Whose problem?



Concluding Questions

Dynamical polarisabilities: Response of internal degrees of freedom to external, real photon field of definite multipolarity and non-zero energy.

Global measure of low energy excitation spectrum/
induced charge distribution caused by interaction with virtual N states.

Contain information about temporal response/dispersive effects of internal nucleon degrees of freedom.

- Can be extracted from Compton amplitudes $\gamma N \to \gamma N$.

  Experimentally not (yet?) directly accessible,

  but dispersion relation extraction uses minimal theoretical prejudice.

- Systematic, model independent microscopic explanation in LO Heavy Baryon Chiral Perturbation Theory & (Modified) Small Scale Expansion.

- Pion production threshold leads to broad enhancement even at low ω.

- Strong ω dependence at low ω in $\beta_{M1}(\omega)$.

- Δ resonance clearly visible in $\beta_{M1}(\omega)$, $\gamma_{M1M1}(\omega)$, $\gamma_{M1E2}(\omega)$:

  para-magnetic $M1 \to M1$, $M1 \to E2$ transitions.

- Compton data on proton understood by dipole polsarisabilities only.

- The Future: Details under investigation.

  Dynamical polarisabilities and deuteron Compton at SAL.

  Other dynamical polarisabilities, e.g. for pions.

  Dynamical generalised polarisabilities.

- **Test your model!**