K - CONDENSATES IN DENSE MATTER

Paolo Bedaque - LBL

- INTRODUCTION
- EFFECTIVE THEORIES AROUND THE FERMI SURFACE:

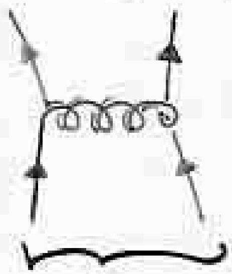
  $p \ll \mu$

  $p \ll \Delta$
- K_0 CONDENSATION, K_+ CONDENSATION, ...
- PHENOMENOLOGICAL CONSEQUENCES:

  TOPOLOGICAL DEFECTS, TRANSPORT, ...

All you need to know about color superconductivity to follow this talk:

— Cooper pairs are "unavoidable"



$\bar{3}$

attractive $\Rightarrow$ $\langle q q \rangle \neq 0$

$$\omega_p = \sqrt{p^2 + \Delta^2}$$

Δ — gap

— in QCD $\quad m_u = m_d = m_s = 0 \Rightarrow$ CFL (Alford, Rajagopal, Wilczek)

$$\langle q_L^{\alpha a i}\, q_L^{\beta b j} \rangle = \langle q_R^{\alpha a i}\, q_R^{\beta b j} \rangle = \Delta\, \epsilon^{\alpha\beta I}\, \epsilon^{abI}\, \epsilon^{ij}$$

α: color, a: flavor, i: Weyl

$\Delta \sim$ 50-100 MeV

$m_u = m_d = 0\ ,\ m_s = \infty \Rightarrow$ 2SC

$$\langle q_L^{\alpha a i}\, q_L^{\beta b j} \rangle = \langle q_R^{\alpha a i}\, q_R^{\beta b j} \rangle = \Delta\, \epsilon^{\alpha\beta\gamma}\, \epsilon^{ab}\, \epsilon^{ij}$$

HIERARCHY ON THE SPECTRUM

$$SU_c(3) \times \underbrace{SU_L(3) \times SU_R(3)}_{U(1)_Q} \times U_B(1) \times \underbrace{U_A(1)}_{\sim (\Lambda_{QCD}/\mu)^5}$$

$$\langle q^{\alpha a i}_{L,R} \, q^{\beta b j}_{L,R} \rangle \sim \epsilon^{\alpha\beta I} \epsilon^{ab I} \epsilon^{ij}$$

$$\underbrace{SU(3)_{C+L+R}}_{\tilde{U}(1)_{\tilde{Q}}} \times \mathbb{Z}_2 \times \mathbb{Z}_2$$

9−1 local symmetries ⇨ 8 massive gluons + 1 massless "photon"

18−8 global symmetries ⇨ 8+1+1 Goldstone bosons

excitation energy

- 2μ — anti-quarks
- $g\mu$ — 8 massive gluons
- Δ — 8+1 gapped quarks
- 8+1+1 GB + 1 "photon"

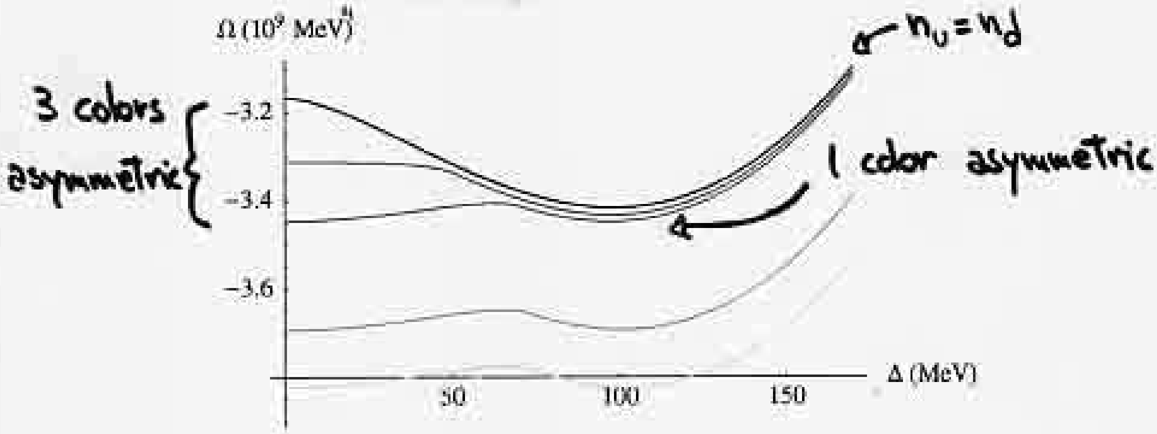
EFT_μ (8 massive gluons, 8+1 gapped quarks, 8+1+1 GB + 1 "photon")

EFT_Δ (8+1+1 GB + 1 "photon")

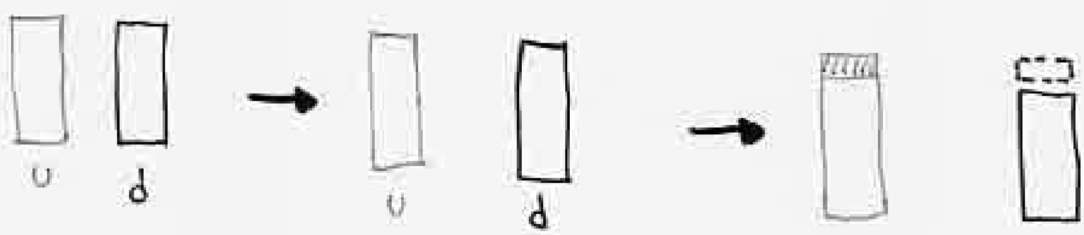
PERTURB CFL ($m_u = m_d = m_s = 0$)

TOWARDS THE REAL WORLD

WHAT DO FLAVOR ASYMMETRIES DO ?



$N_F = 2 \Rightarrow$ blue and red are paired, grens are not



$\mu_u = \mu_d = \mu$ $\quad$ $\mu_u + \delta$ $\quad \mu_d = \mu - \delta$ $\quad$ $\mu_u = \mu + \delta$ $\quad \mu_d = \mu - \delta$

$\delta < \Delta$ $\qquad$ $\delta > \Delta$

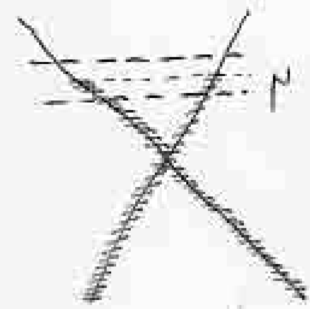
RIGIDITY

EFT_μ $(p \ll \mu)$ (D.K. Hong '99)

$$(p_0+\mu)\,\psi_\pm = \pm |p|\,\psi_\pm$$

energy measured from the Fermi surface

$$\psi_\pm = \frac{1 \pm \vec{\alpha}\cdot\hat{p}}{2}\,\psi$$



$\psi_+ :\quad p_0 = -\mu + |p| \;\Rightarrow$ small excitations $p_0 \simeq 0$

$\psi_- :\quad p_0 = -\mu - |p| \;\Rightarrow$ p_0 large, integrate ψ_- out

$$S = \int d^4x\; \bar{\psi}\,(i\not{D} + \mu\gamma^0 - \mu_e Q\gamma^0)\,\psi - \bar{\psi}_L M \psi_R - \bar{\psi}_R M^\dagger \psi_L + \cdots$$

$$\Downarrow$$

$$S = \int \frac{d^4p}{(2\pi)^4}\; \psi^\dagger_{+L}(p_0 - p + \mu - v.A)\,\psi_{+L} + \psi^\dagger_{L+}\Big(\underbrace{-\mu_e Q - \frac{MM^\dagger}{2\mu}}_{\mu_e \text{ is like } M}\Big)\psi_{L+} + L \to R + \cdots$$

approximate symmetry $O(V_\mu)$

$$\psi_L \to L(t)\,\psi_L$$
$$\psi_R \to R(t)\,\psi_R$$

$$-\mu_e - \frac{MM^\dagger}{2\mu} \to i L \partial_0 L^\dagger + L\Big(-\mu_e - \frac{MM^\dagger}{2\mu}\Big)L$$

$$-\mu_e - \frac{M^\dagger M}{2\mu} \to i R \partial_0 R^\dagger + R\Big(-\mu_e - \frac{M^\dagger M}{2\mu}\Big)R$$

EFT$_\Delta$ $(p \ll \Delta)$

$$\mathcal{L} = \frac{f^2}{4} \mathrm{Tr}\left(\nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2 \nabla\Sigma \cdot \nabla\Sigma^\dagger \right) + 2A\,|M|\,\mathrm{Tr}\left(M^{-1} \Sigma + h.c. \right)$$
$$+ \ldots$$

$$\nabla_0 \Sigma = \partial_0 \Sigma + i\left(\mu_e Q + \frac{MM^\dagger}{2\mu} \right) \Sigma - i \Sigma \left(\mu_e Q + \frac{M^\dagger M}{2\mu} \right)$$

masses are like chemical pot. for flavor

generic Term:

$$\sim f^2 \Delta^2 \left(\frac{\partial_0 - i\mu_e Q - i MM^\dagger/2\mu}{\Delta} \right)^n \left(\frac{\vec{\nabla}}{\Delta} \right)^m \left(\frac{MM}{\mu^2} \right)^p \left(\frac{\mu_e Q}{\mu} \right)^q$$

- NO TERM LINEAR ON M : $\mathbb{Z}_2$
- LOOPS VERY SUPPRESSED $\sim \frac{1}{4\pi f_\pi} \sim \frac{1}{\mu^2}$
- TERMS SUPPRESSED BY $\frac{1}{\Delta}$ OR $\frac{1}{\mu}$: TWO UV SCALES

MESON MASSES

$$m_{\pi^\pm} = \mp \frac{m_d^2 - m_u^2}{2\mu} + \sqrt{\frac{4A}{f^2} m_s (m_u + m_d)}$$

$$m_{K^\pm} = \mp \frac{m_s^2 - m_u^2}{2\mu} + \sqrt{\frac{4A}{f^2} m_d (m_u + m_s)}$$

$$m_{K^0} = \mp \frac{m_s^2 - m_d^2}{2\mu} + \sqrt{\frac{4A}{f^2} m_u (m_d + m_s)}$$

$$A \sim \frac{f^2 \Delta^2}{\mu^2}$$

"strangeness chemical potential term"

m_s^4

$$\underbrace{\left(\frac{M M^\dagger}{\mu}\right)^2}_{m^4/\mu^2} \sim \frac{f^2\Delta^2}{\mu^2} \underbrace{\det M \; M^{-1}}_{\Delta^2 M^2/\mu^2}$$

$m_s \, m_{light}$

$$\frac{m^4}{\mu^2} \sim \frac{\Delta^2 M^2}{\mu^2} \Rightarrow M \sim \Delta \Rightarrow m_K \sim \frac{\Delta M}{\mu} \ll \Delta$$

$$\boxed{m_{K^0} \text{ CAN BE NEGATIVE: } K_0 \text{ CONDENSATION}}$$

GROUND STATE

$$\Sigma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & e^{i\theta}\sin\alpha \\ 0 & e^{-i\theta}\sin\alpha & \cos\alpha \end{pmatrix}$$

$$V = -\frac{f^2}{8\mu^2}\left(m_s^4\cos^2\alpha + 2m_s^2\underbrace{\hat{m}^2}_{\frac{m_u+m_d}{2}}\sin^2\alpha\right) - 4A\cos\alpha\, m_s\hat{m}$$

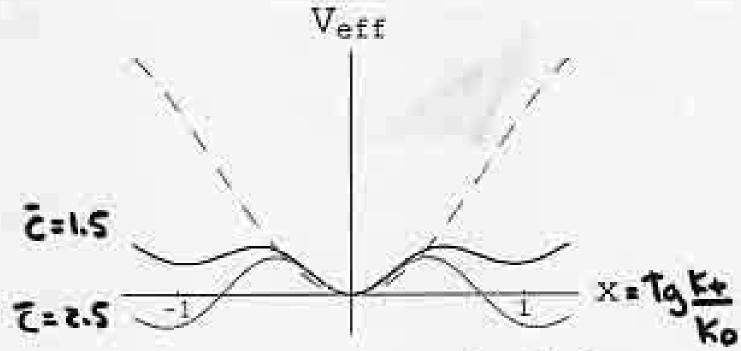
$$\cos\alpha = -\frac{16A\mu^2 m_u}{f^2 m_s^3} \sim \underbrace{\frac{A\, m_s m_u}{f^2}}_{\text{"}m_{K_0}^2\text{"}} \underbrace{\frac{\mu^2}{m_s^4}}_{\text{"}1/\mu_s^2\text{"}}$$

$\simeq 0.5$ (Taking perturbative calculations)

K_0 is favored ove K_+ because

- $m_d > m_u$
- e.m. mass
- $\langle K_+\rangle \neq 0 \Rightarrow e^-$

all small isospin breaking effects

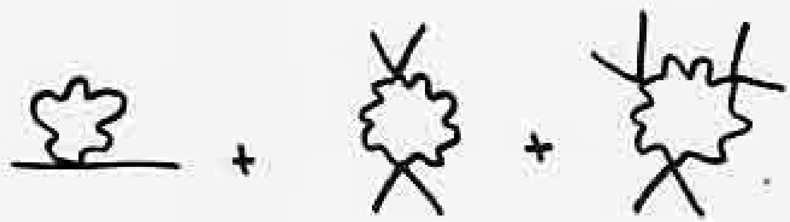


INSULATOR

OR

SUPERCONDUCTOR ?

$f = 3 f_{pert}$, $\Delta = 50$ MeV , $\xi = -0.5$

V =  + . . .

e.m. mass

$\sim b\, f^2\, \mathrm{tr}[Q,\Sigma^\dagger][\Sigma,Q]$ $\qquad$ $\sim c\, f^4 \left(\mathrm{tr}[Q,\Sigma^\dagger][\Sigma,Q]\right)^2$

$b(\sim 2\Delta) \sim \frac{1}{8\pi}\left(\frac{2v^2}{\sqrt{\epsilon}} + \frac{1}{v\epsilon^2}\right)$ $\qquad$ $c(\sim 2\Delta) \sim \frac{1}{16}\left(\frac{2v^4}{\sqrt{\epsilon}} + \frac{v}{\epsilon^2}\right)$

Coleman-Weinberg mechanism condenses K_+ for

"HALF" OF PARAMETERS w/ $\mu \sim 10$ GeV

"SOME" OF PARAMETERS w/ $\mu \sim 500$ MeV

K_0 condensation is like the SM

w/ isospin $SU_I(2) \times U_Y(1) \longrightarrow U_{\tilde{Q}}(1) \Rightarrow$ 1 linear + 1 quad. GB

w/o isospin $U_u(1) \times U_d(1) \longrightarrow U_{\tilde{Q}}(1) \Rightarrow$ 1 linear GB

(Kaplan & Reddy, '02)

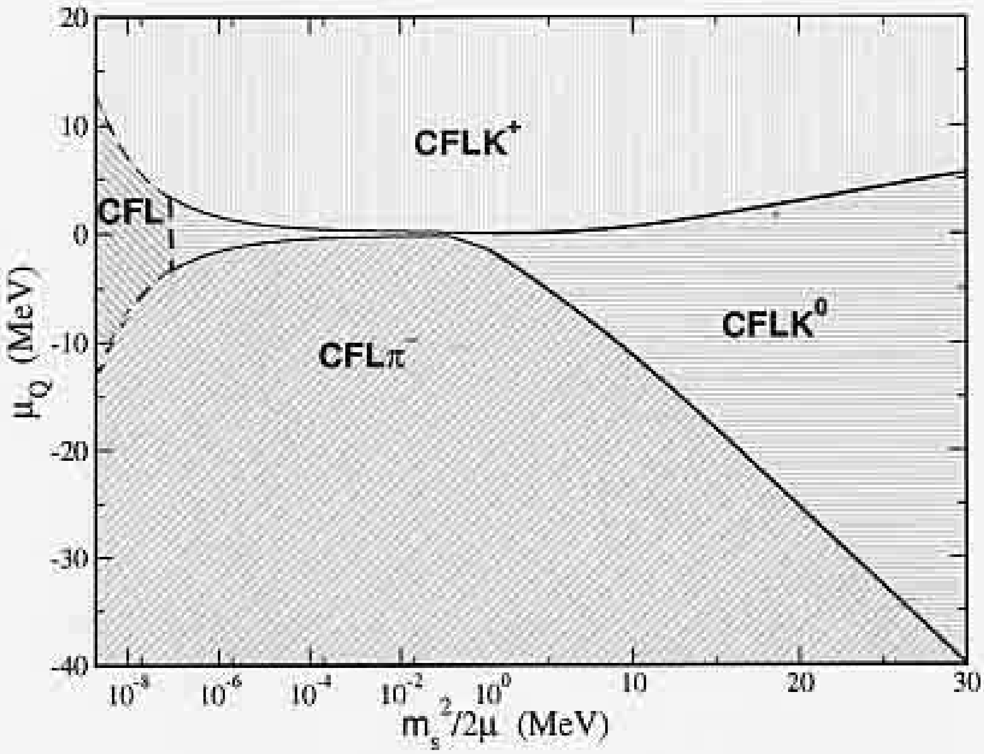


Figure 1: Meson condensed phases in the neighborhood of the symmetric CFL state are shown in the $(m_s^2/2\mu) - \mu_Q$ plane, where m_s is the strange quark mass (set to 150 MeV), μ is the quark number chemical potential, and μ_Q is the chemical potential for positive electric charge. At five times nuclear density $\mu \sim 400$ MeV and $(m_s^2/2\mu) \sim 25$ MeV. Solid and dashed lines indicate first- and second-order transitions respectively.

THE SERIOUS WAY OUT

$U_A(1)$ breaking not so suppressed at $\mu \simeq 500$ MeV

$B\, \mathrm{Tr}(M\Sigma + M\Sigma^{\dagger})$

dominated by large instantons

shamelessly using the "perturbative calculation"

$7\ \mathrm{MeV} < \Delta m_K < 90\ \mathrm{MeV}$ (Schäfer '02) (Manuel, Tytgat '00)

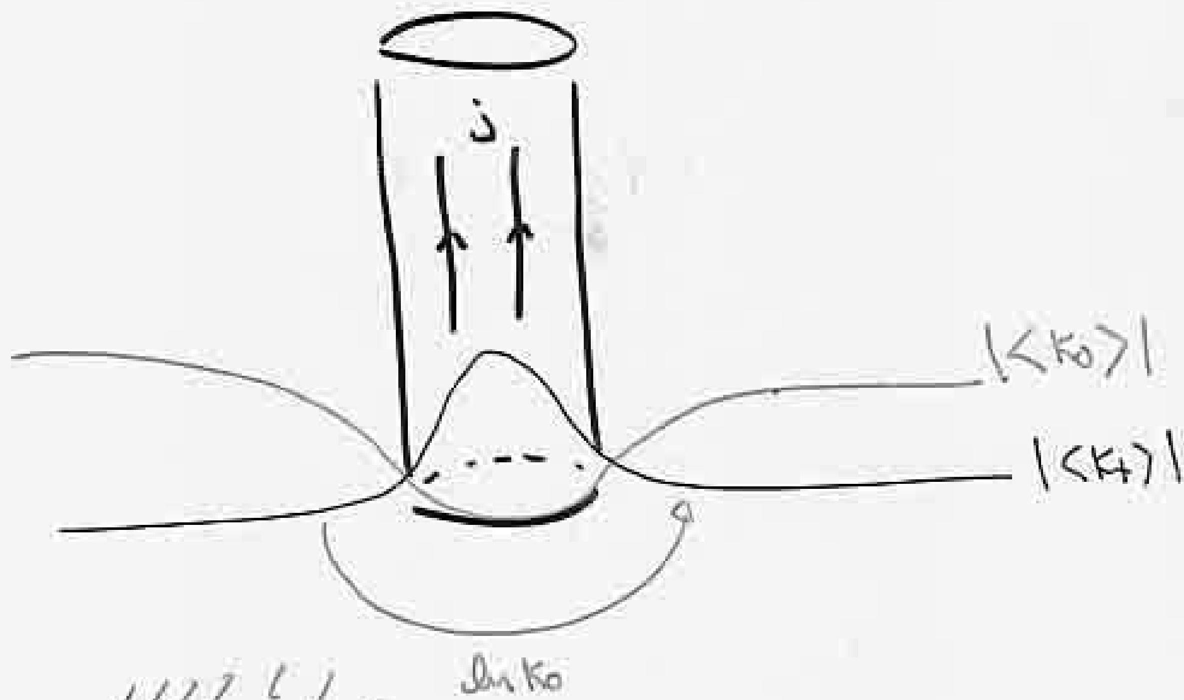
2-loops β function, screening mass

1-loop β function,

(Kaplan, Reddy '02, Zhitnitsky, Buckley, '02)

$U_Y(1)$ BROKEN ⇒ GLOBAL STRINGS ⇒ $T \sim \ln R$

SUPERCONDUCTING STRING



⇒ FINITE ENERGY AND STABLE

('Son 01)

METASTABLE DOMAIN WALL

electroweak mass

$K_0 = A e^{i\phi}$ $\qquad \phi(z) = 4\, \mathrm{arctg}\, e^{mz/v}$

SUMMARY

- antiquarks are irrelevant $\Rightarrow$ m_s^2/μ like a μ_s

  $\Rightarrow$ condensation of strange mesons

  $$\underbrace{\frac{m_s^2}{\mu}}_{\text{“}\mu_s\text{”}} \sim \underbrace{\frac{\Delta}{\mu}\sqrt{m_u m_s}}_{\text{“}m_{K^0}\text{”}} \Rightarrow m_s \sim m^{1/3}\,\Delta^{2/3}$$

- K_0 condensed phase has very different properties from CFL

- all kinds of interesting phases, solitons

- instantons can still destroy all this

- actual implications to neutron/strange stars/nuggets just starting

- cute, probably right and might even be usefull