The phases of quark matter

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Reviews:

M. Alford: `hep-ph/0102047` K. Rajagopal, F. Wilczek: `hep-ph/0011333`

I. Color superconductivity

Low temperatures and densities: confined/broken chiral symmetry phase of QCD.

High temperatures: quark-gluon plasma (QGP)

- chiral symmetry restored
- deconfinement
- signatures sought at heavy-ion colliders

High densities: color superconductivity

quarks *pair* in color non-singlets. Various phases:

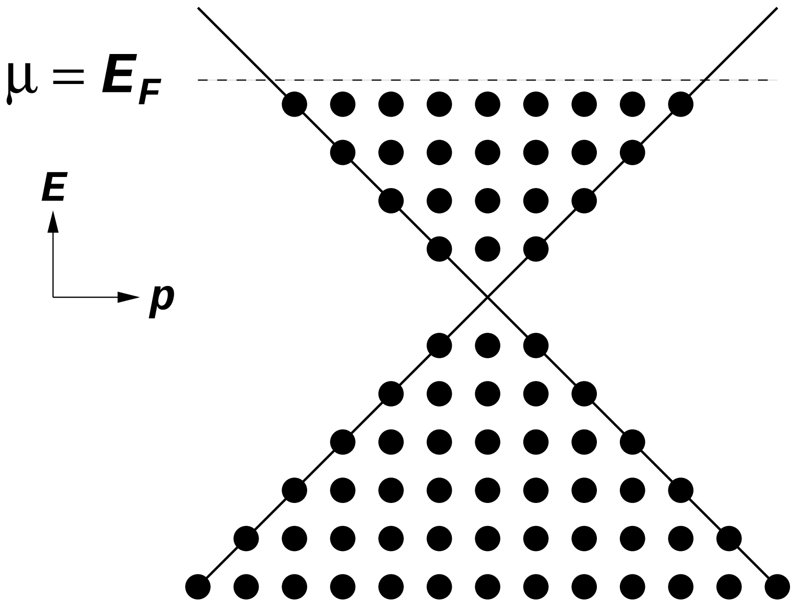
$N_f = 3$: CFL, chiral symmetry and baryon number broken.

$N_f = 2$: 2SC, chiral symmetry restored, baryon number unbroken.

What sort of quark matter do we expect in compact stars?

Quarks at very high density

At sufficiently high density and low temperature, there is a Fermi sea of almost free quarks.



$$F = E - \mu N$$

But quarks have attractive QCD interactions.

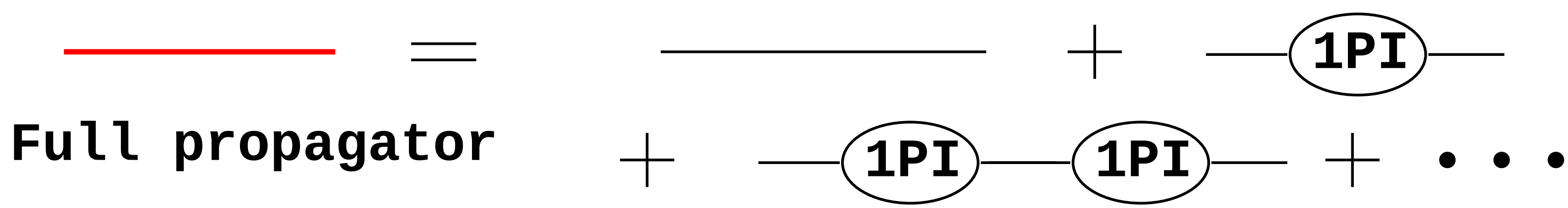
Any attractive quark-quark interaction causes pairing instability of the Fermi surface. This is the Bardeen-Cooper-Schrieffer (BCS) mechanism of superconductivity.

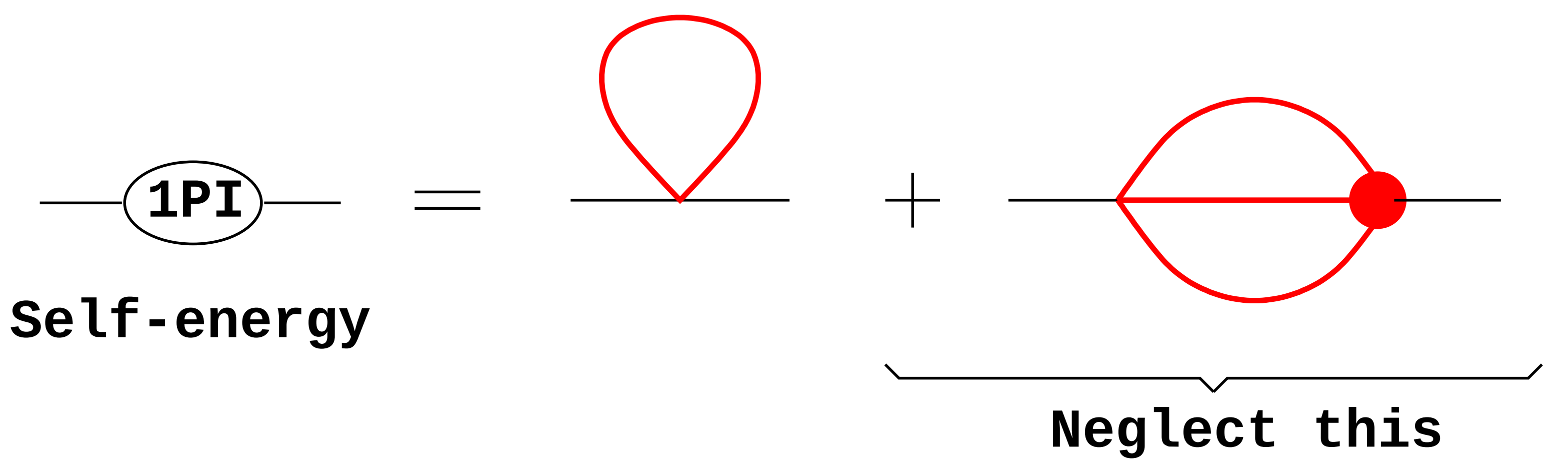
$$\langle qq \rangle \neq 0$$

Gap equation in field theory

The field-theoretic way to look for spontaneous symmetry breaking is to make an ansatz for the self-energy, and solve Schwinger-Dyson equations.

This is diagrammatic, but *non-perturbative*.

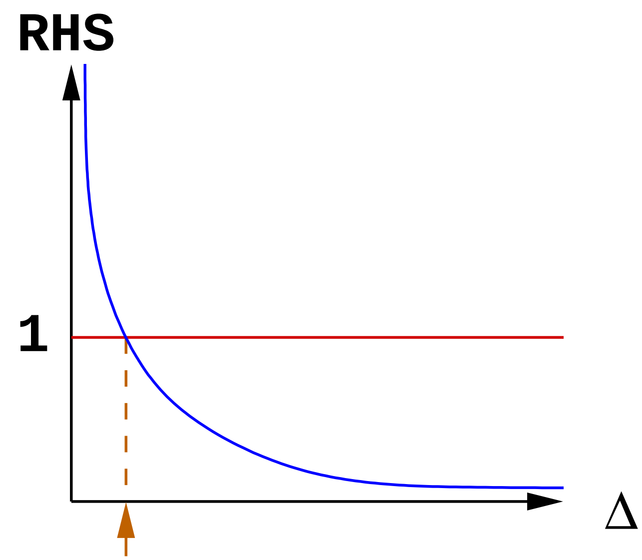




Gap equation

$$1 = \frac{8K}{\pi^2}\int_0^{\Lambda} p^2 dp \left\{ \frac{1}{\sqrt{\Delta^2 + (p-\mu)^2}} \right\}$$

$$\Delta = \langle qq \rangle_{1PI}$$



Note BCS divergence as $\Delta \to 0$: there is *always* a solution, for any interaction strength K and chemical potential μ.

Roughly,

$$1 \;\sim\; K\mu^2 \ln(\Lambda/\Delta)$$

$$\Rightarrow \Delta \;\sim\; \Lambda \exp\left(-\frac{1}{K\mu^2}\right)$$

Superconducting gap is non-perturbative.

II. Different pairing patterns

Three massless flavors: Color-flavor locking (CFL)

Equal number of colors and flavors gives a special pattern of symmetry breaking:

$$\langle q_i^\alpha q_j^\beta \rangle \sim \delta_i^\alpha \delta_j^\beta + \kappa\, \delta_j^\alpha \delta_i^\beta$$

color α, β

flavor i, j

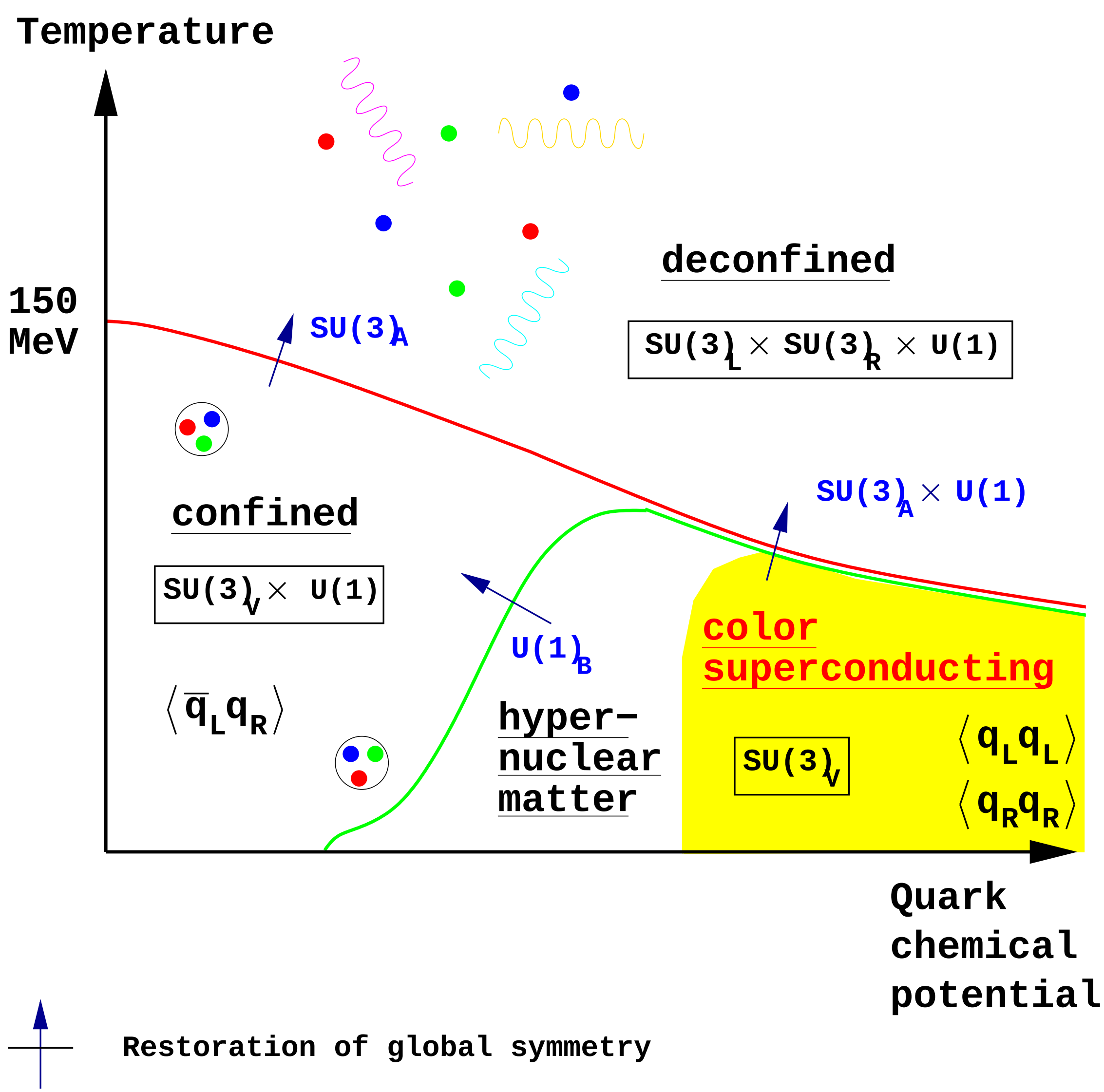
This is invariant under equal and opposite rotations of color and (vector) flavor

$$SU(3)_{\text{color}} \times \underbrace{SU(3)_L \times SU(3)_R}_{\supset U(1)_Q} \times U(1)_B \rightarrow \underbrace{SU(3)_{C+L+R}}_{\supset U(1)_{\tilde{Q}}} \times \mathbb{Z}_2$$

- Breaks chiral symmetry, but *not* by a $\langle \bar{q} q \rangle$ condensate.
- There need be no phase transition between the low and high density phases: ("quark-hadron continuity")
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.

Color-flavor locking phase diagram

(three massless flavors)



Two-flavor color superconductor (2SC)

$$\langle q_i^\alpha q_j^\beta \rangle \sim \varepsilon_{ij} \varepsilon^{\alpha\beta 3}$$

color α, β, flavor i, j

This is a flavor singlet, color $\bar{\mathbf{3}}$.

$$SU(3)_{\text{color}} \times U(1)_Q \times SU(2)_L \times SU(2)_R$$
$$\rightarrow \; SU(2)_{\text{color}} \times U(1)_{\tilde{Q}} \times SU(2)_L \times SU(2)_R$$

- No global symmetries are broken, so no order parameter.
- Chiral symmetry is restored at high density.
- Unbroken "rotated" electromagnetism, $\tilde{Q}$, photon-gluon mixture.
- Unbroken "rotated" baryon number $\tilde{B} = \tilde{Q} + I_3$.

III. Compact stars

Where in the universe is color-superconducting quark matter most likely to exist? In compact stars.

A quick history of a compact star.

A star of mass $M \gtrsim 10 M_\odot$ burns Hydrogen by fusion, ending up with an Iron core. Core grows to Chandrasekhar mass, collapses $\Rightarrow$ supernova. Remnant is a compact star:

mass	radius	density	initial temp
$\sim 1.4 M_\odot$	$\mathcal{O}(10\text{ km})$	$\geqslant \rho_{\text{nuclear}}$	$\sim 30\text{ MeV}$

The star cools by neutrino emission for the first million years.

Signatures of color superconductivity in compact stars

Transport properties, mean free paths, conductivities, viscosities, etc.

1. Glitches and crystalline ("LOFF") pairing
2. Cooling by neutrino emission, neutrino pulse at birth
3. r-mode instability

Equation of state

Pressure of quark matter relative to hadronic vacuum

$$p \sim \mu^4 + \Delta^2 \mu^2 - B$$

If bag constant is large enough to bring quark matter close to stability, a superconducting gap Δ may have large effects.

Quark matter in compact stars

- Weak equilibrium

$$
\begin{aligned}
u &\rightarrow d\, e^+\, \bar{\nu} & \mu_u &= \bar{\mu} - \tfrac{2}{3}\mu_e \\
u &\rightarrow s\, e^+\, \bar{\nu} & \mu_d = \mu_s &= \bar{\mu} + \tfrac{1}{3}\mu_e
\end{aligned}
$$

- Electromagnetic neutrality

$$\tfrac{2}{3}N_u - \tfrac{1}{3}N_d - \tfrac{1}{3}N_s - N_e = 0$$

But there may be a globally neutral mixture of positive nuclear matter with negative quark matter

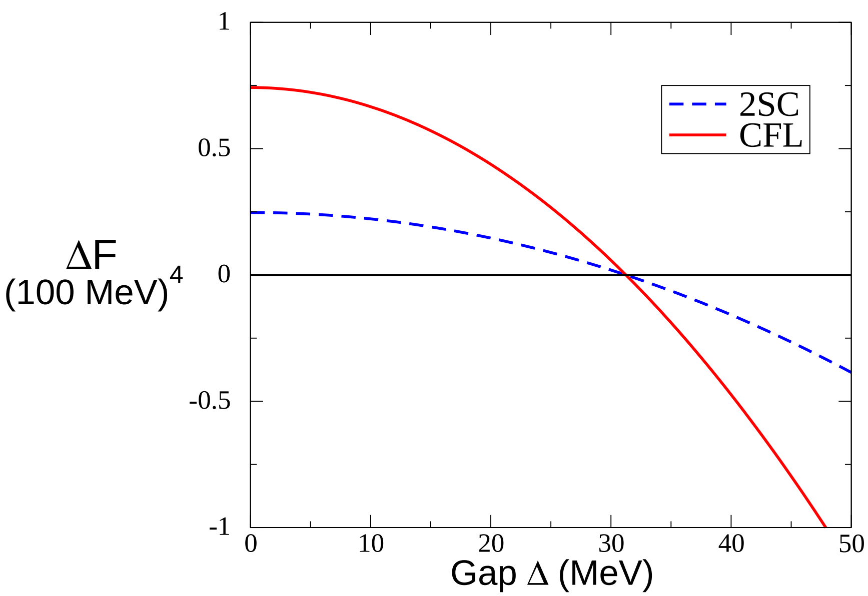
- Color neutrality. In unitary gauge, number of red, green, blue quarks must be the same. The cost of projecting to a color singlet is then negligible.

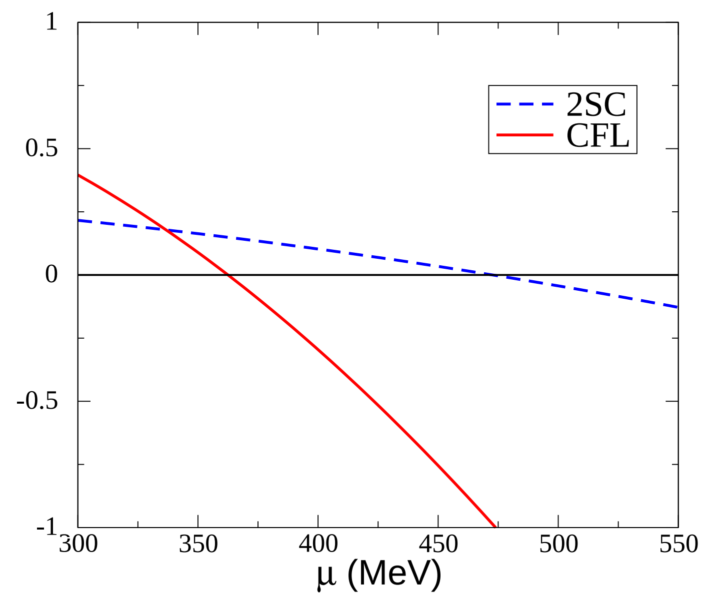
$$\langle Q_3 \rangle = \langle Q_8 \rangle = 0, \quad T_3 = \mathrm{diag}(1, -1, 0), \quad T_8 = \mathrm{diag}(1, 1, -2)$$

Does the 2SC phase occur in compact stars?

Electric/color neutrality imposes a free energy cost. Expanding free energy to $(M_s/\mu)^4$ and $(\Delta/\mu)^2$, we find that 2SC never occurs.

Plot free energy $\Delta F = F - F_{\text{unpaired}}$:





Fixed $M_s = 250$ MeV

$$M_s \quad = \quad 275 \text{ MeV}, \quad 250 \text{ MeV in CFL}$$

$$\Delta \quad = \quad 40 \text{ MeV}, \quad 30 \text{ MeV in CFL}$$

"Unpaired" region will have crystalline pairing or $J \neq 0$ self-pairing.

Neutral unpaired, CFL, and 2SC free energies:

$$\Omega_{\text{CFL}}^{\text{neutral}} = \Omega_{\text{unpaired}}^{\text{neutral}} + 18\frac{1}{96\pi^2}\left({M_s}^4 - 16\Delta^2\mu^2\right)$$
$$\Omega_{\text{2SC}}^{\text{neutral}} = \Omega_{\text{unpaired}}^{\text{neutral}} + 6\frac{1}{96\pi^2}\left({M_s}^4 - 16\Delta^2\mu^2\right)$$

Neutral paired is favored over neutral unpaired when $\Delta > \dfrac{{M_s}^2}{4\mu}$.

So evaluating Ω to order ${M_s}^4$ and to order Δ^2 was consistent.

$$\Omega_{\text{unpaired}}^{\text{neutral}} = -\frac{3\mu^4}{4\pi^2} + \frac{3{M_s}^2\mu^2}{4\pi^2} - \frac{21 - 36\log(M_s/2\mu)}{96\pi^2}{M_s}^4$$
$$\Omega_{\text{2SC}}^{\text{neutral}} = -\frac{3\mu^4}{4\pi^2} + \frac{3{M_s}^2\mu^2}{4\pi^2} - \frac{15 - 36\log(M_s/2\mu)}{96\pi^2}{M_s}^4 - 1\frac{\Delta^2\mu^2}{\pi^2}$$
$$\Omega_{\text{CFL}}^{\text{neutral}} = -\frac{3\mu^4}{4\pi^2} + \frac{3{M_s}^2\mu^2}{4\pi^2} - \frac{3 - 36\log(M_s/2\mu)}{96\pi^2}{M_s}^4 - 3\frac{\Delta^2\mu^2}{\pi^2}$$

The $\mathcal{O}(M_s/\mu)^4$ terms differ due to the imposition of neutrality, and costs of locking the Fermi surfaces together before pairing.

The coefficient of ${M_s}^4/(96\pi^2)$ starts off as -27, and then gets the following contributions:

pairing	locking	neutralizing	total
unpaired	0	$+6$	$+6$
2SC	0	$+12$	$+12$
CFL	$+20$	$+4$	$+24$

Relative to unpaired, CFL pays $+18$ and 2SC pays $+6$, a factor of 3, which happens to be the same as the factor by which their diquark condensate binding energies differ.

CFL free energy, to order $(\Delta/\mu)^2$

$$\Omega_{\rm CFL} = \frac{1}{\pi^2}\sum_{i=1}^{9}\int_0^{p_{F,i}^{\rm common}}\left(\sqrt{p^2+M_i^2}-\mu_i\right)p^2dp - \frac{3}{\pi^2}\Delta^2\mu^2 \ ,$$

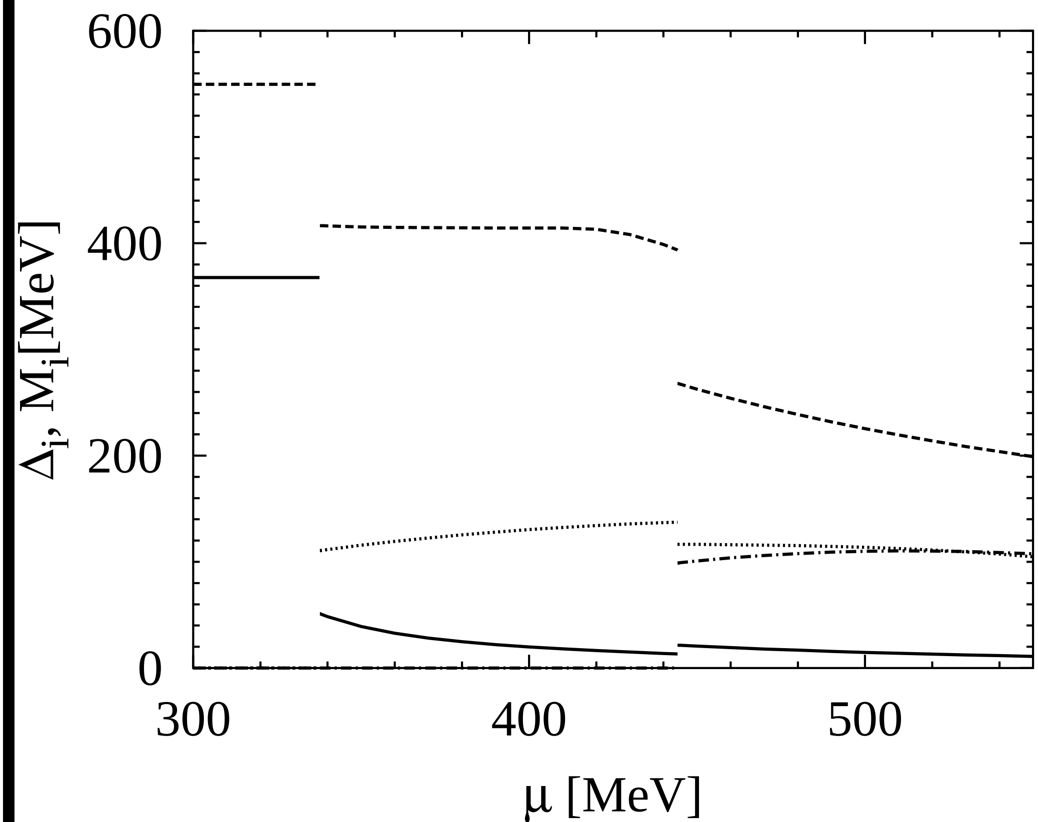
Introducing strange quark mass M_s and chem pots for gauged charges, and assuming $(M_s/\mu)^2 \lesssim \Delta/\mu$,

$$\begin{aligned}\Omega_{\rm CFL} \;=\; & -\frac{3\mu^4}{4\pi^2}+\frac{3{M_s}^2\mu^2}{4\pi^2}-\frac{1}{96\pi^2}\Big[\Big(16\mu_8-8\mu_e\Big){M_s}^2\mu \\ & +\Big(16{\mu_e}^2+12{\mu_3}^2+16{\mu_8}^2-24\mu_3\mu_e-16\mu_e\mu_8\Big)\mu^2 \\ & +7{M_s}^4-36{M_s}^4\log(M_s/2\mu)\Big]-\frac{3\Delta^2\mu^2}{\pi^2}\end{aligned}$$

Fix μ_e, μ_3, μ_8 by electromagnetic and color neutrality,

$$\frac{\partial\Omega_{\rm CFL}}{\partial\mu_e}=\frac{\partial\Omega_{\rm CFL}}{\partial\mu_3}=\frac{\partial\Omega_{\rm CFL}}{\partial\mu_8}=0$$

M_s is lower in CFL: evidence from an NJL model



Solid: $M_{u,d}(\mu)$; dashed: $M_s(\mu)$; dotted: $\langle ud \rangle$; dash-dotted: $\langle us \rangle$.

We see that M_s drops by $\sim 25\%$ from 2SC to CFL, and diquark condensates drop slightly too.

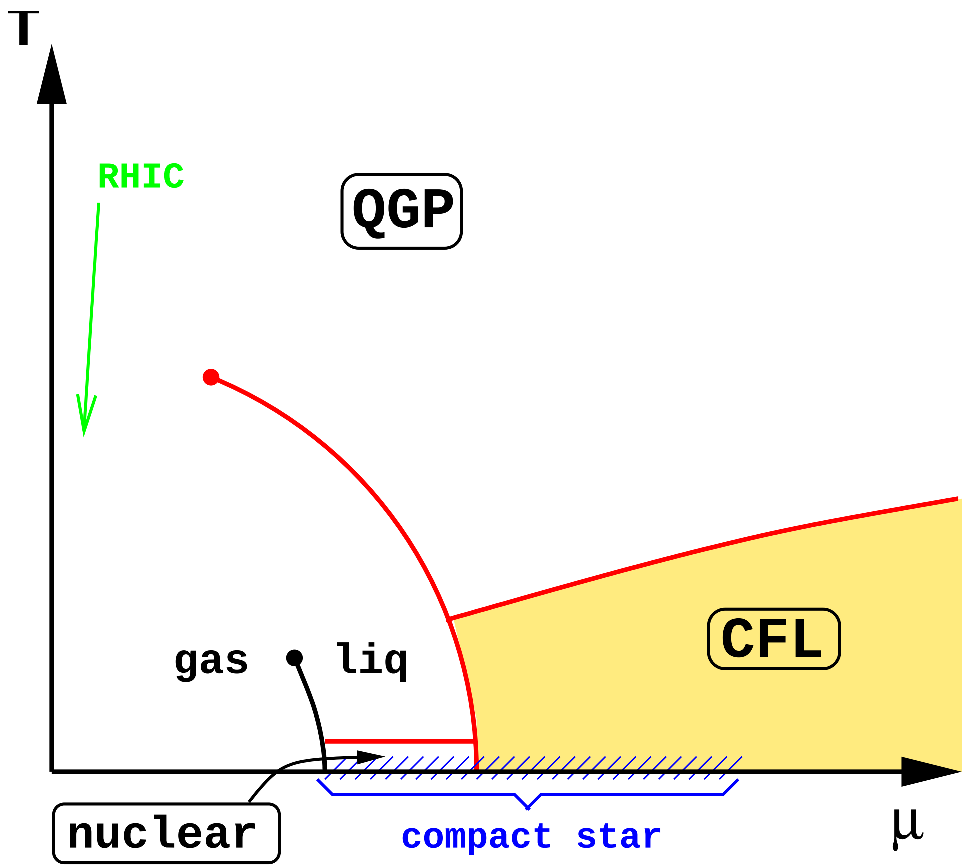
(Buballa and Oertel, `hep-ph/0202098`).

NJL model calculation, including 3 flavor instanton vertex, but not imposing color/electric neutrality.

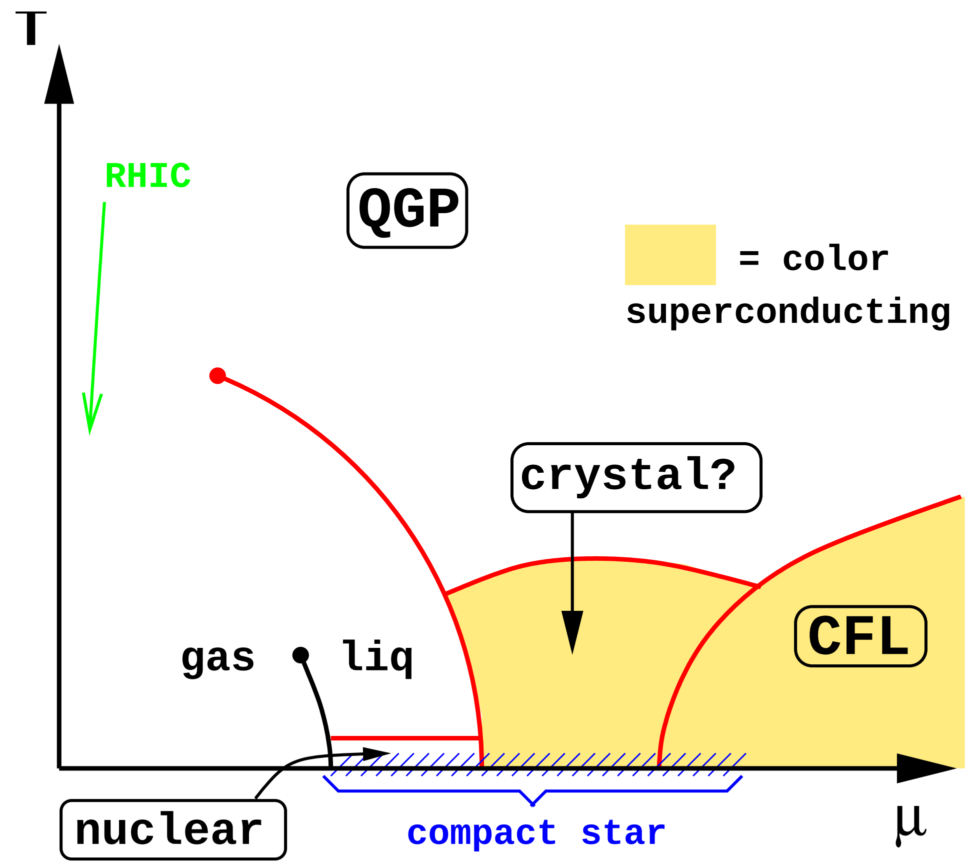
Things we have ignored.

1. If M_s is bigger in 2SC than CFL, then the binding energy E_C of the chiral condensate should be bigger in 2SC.
   Could this re-open the 2SC window? No.
   $E_C \sim (m_s, M_s, \langle \bar{q} q \rangle)^4$, so it is an $\mathcal{O}({M_s}^4)$ term, dominated by the $\mathcal{O}({M_s}^2)$ penalty.
   It would be good to perform a full NJL calculation with coupled gap equations for the chiral and diquark condensates.
2. Perturbative corrections to the free energy: these are the same for all quark matter phases.
3. Kaon condensation: lowers CFL free energy at $\mathcal{O}({M_s}^4)$.
4. Possible μ- and M_s-dependence of the gap Δ.

Possible bulk nuclear/quark matter phase diagrams



Nuclear to CFL directly

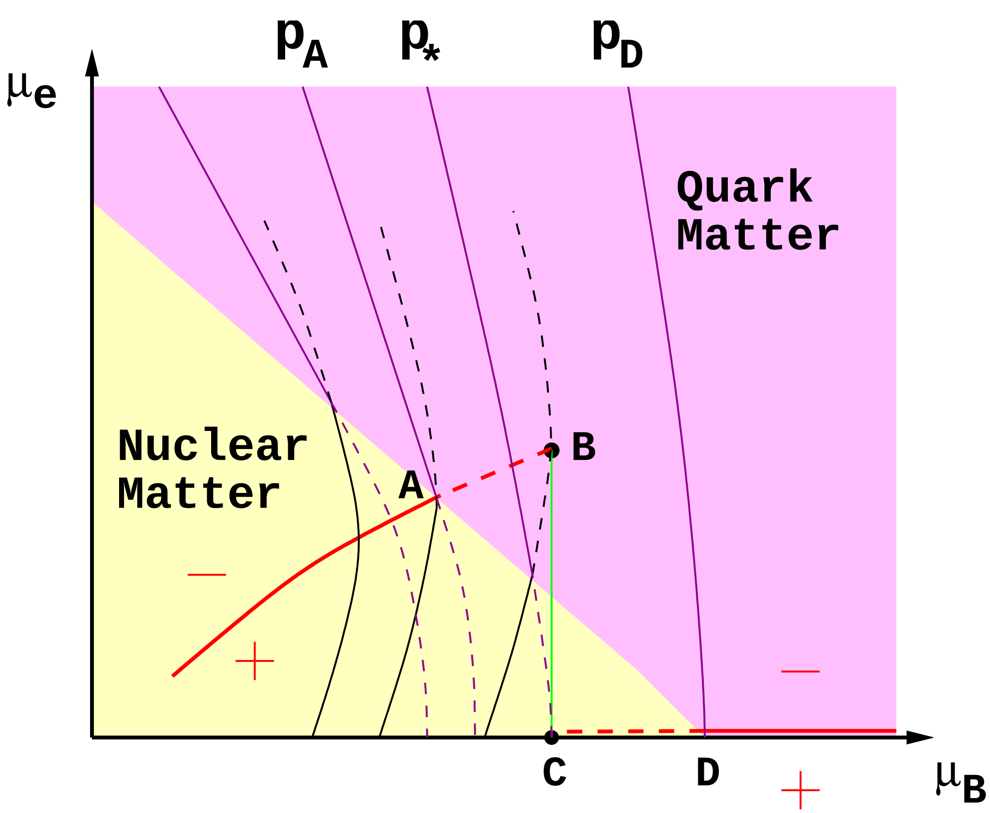


Intermediate quark matter phase between nuclear and CFL

Mixed NM-CFL phase in compact stars

Glendenning, Phys. Rev. D46, 1274 (1992)

$$Q = \left.\frac{\partial p}{\partial \mu}\right|_{\mu_B}$$



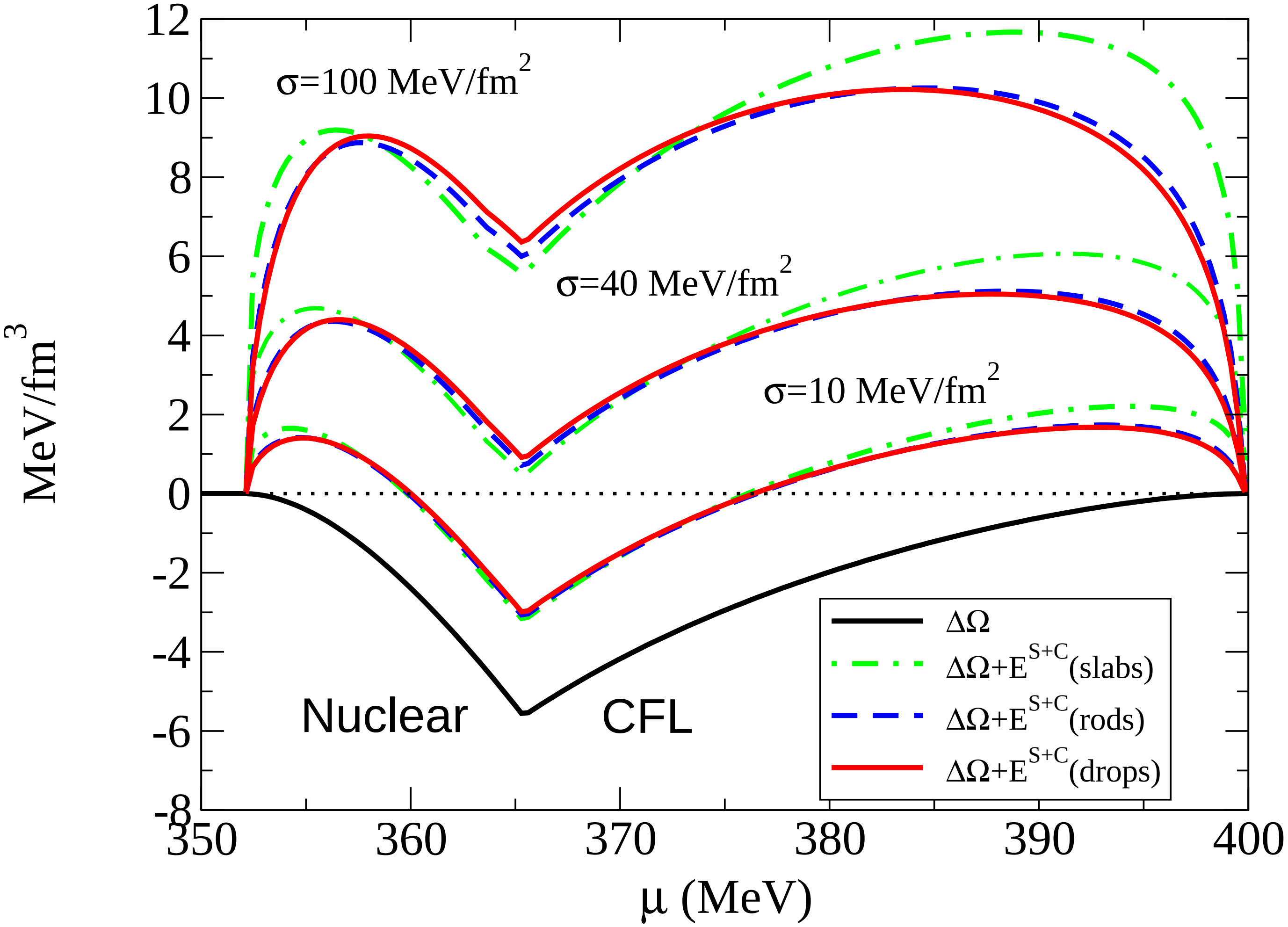
(1) Impose local neutrality: NM to CFL transition at one radius, $A \rightarrow B \rightarrow C \rightarrow D$.

(2) Only impose global neutrality: NM to CFL transition over a range of radii, following *mixed phase* co-existence line $A \rightarrow D$.

Mixed phase is favored if surface tension is small, so positive NM and negative CFL can comingle.

Sharp interface or mixed phase?

Free-energy advantage of sharp transition over mixed phase, $F_{\mathsf{mixed}} - F_{\mathsf{sharp}}$.



For $\sigma \gtrsim 40$ MeV, the sharp interface is favored.

IV. Looking forward

- Compact-star phenomenology:
  - Crystalline phase and glitches
  - Nuclear-quark interface: mixed phase
  - conductivity and emissivity (neutrino cooling)
  - shear and bulk viscosity (r-mode spin-down)
- Other phenomenology:
  - Diquark condensate model of *zero density* confining phase.
  - Role of quark pairing in heavy-ion collisions.
- Other questions:
  - "Kaon" condensation in CFL phase
  - Better weak-coupling calculations, include vertex corrections
  - Go beyond mean-field, include fluctuations.